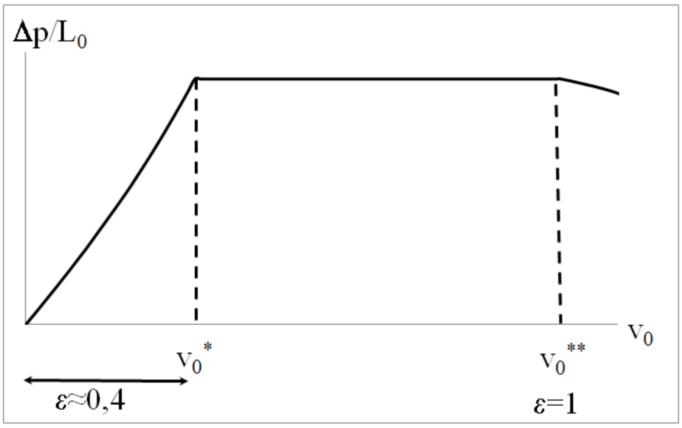
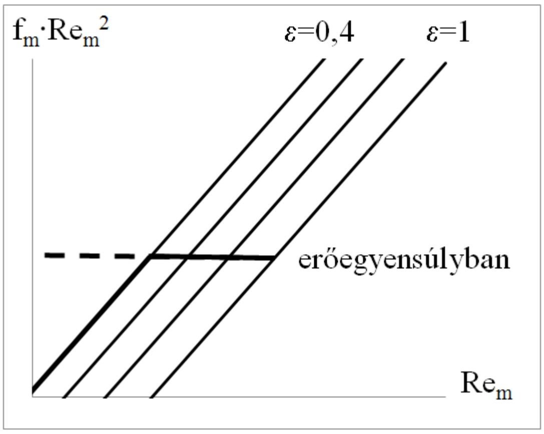
FLUIDIZATION

Fluidization plot (log-log)



u_0: velocity calculated assuming empty tube

$Re_m - f_m \cdot Re_m^2$ diagram (log-log)



erőegyensúlyban: in the case of balanced forces

Definition of the modified Re number: $Re_m = \frac{d_p \cdot u_0 \cdot \rho}{\eta}$

Balanced forces (during fluidization):

$$f_m \cdot Re_m^2 = \frac{d_p^3 \cdot (\rho_p - \rho) \cdot \rho \cdot g}{2 \cdot \eta^2}$$

Calculation of pressure drop

General formula

$$\Delta p = 4 \cdot f_m \cdot \frac{L_0}{d_p} \cdot \frac{u_0^2 \cdot \rho}{2}$$

Ergun formula for packing in rest, $\varepsilon \leq 0.5$:

$$\Delta p_E = \frac{1-\varepsilon}{\varepsilon^3} \cdot \frac{L}{d_p} \cdot \left[1.75 + \frac{150 \cdot (1-\varepsilon)}{Re_m}\right] \cdot u_0^2 \cdot \rho$$

During fluidization:

$$\Delta p_{grid} = L \cdot (1-\varepsilon) \cdot (\rho_p - \rho) \cdot g = L_0 \cdot (\rho_p - \rho) \cdot g$$

Note 1.

For solving the problems one has to determine if there is fluidization of not.

a) Based on the chart $Re_m - f_m \cdot Re_m^2$
b) Based on the relation of Δp_E and Δp_{grid}

If	$\Delta p_E < \Delta p_{grid}$	→	static region (rest)	$\Delta p = \Delta p_E$
If	$\Delta p_E > \Delta p_{grid}$	→	fluidization region	$\Delta p = \Delta p_{grid}$

Note 2.

In case of fixed packing:

- no fluidization
- Constant ε curve in the chart $Re_m - f_m \cdot Re_m^2$
- Ergun-formula can be applied everywhere.

Problem 1

A column of diameter 260 mm is filled with spherical ceramic packing (ρ_p = 2400 kg/m^3, ε = 0.4) of diameter 2 mm up to the height of 2.4 m. Air stream with pressure 1 bar, temperature 20°C (η = 0.018 mPas), and flow rate 113 kg/h enters at the bottom of the column.
How much is the pressure drop over the packing?

Solution
Air density according the ideal gas low

$$\rho = \frac{p \cdot M_{air}}{R \cdot T} = \frac{10^5 \mathrm{Pa} \cdot 29 \frac{\mathrm{g}}{\mathrm{mol}}}{8.314 \frac{\mathrm{J}}{\mathrm{mol} \cdot \mathrm{K}} \cdot 293\mathrm{K}} = 1190 \frac{\mathrm{g}}{\mathrm{m}^3} = 1.19 \frac{\mathrm{kg}}{\mathrm{m}^3}$$

Air velocity

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{113 \frac{\mathrm{kg}}{\mathrm{h}}}{1.19 \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 3600 \frac{\mathrm{s}}{\mathrm{h}}} = 2.64 \cdot 10^{-2} \frac{\mathrm{m}^3}{\mathrm{s}}$$

$$u_0 = \frac{\dot{V}}{A_{tube}} = \frac{\dot{V}}{\frac{(D_{tube})^2 \cdot \pi}{4}} = \frac{2.64 \cdot 10^{-2} \frac{\mathrm{m}^3}{\mathrm{s}}}{\frac{(0.26\mathrm{m})^2 \cdot \pi}{4}} = 0.497 \frac{\mathrm{m}}{\mathrm{s}}$$

$$\mathrm{Re}_m = \frac{d_p \cdot v_0 \cdot \rho}{\eta} = \frac{2 \cdot 10^{-3} \mathrm{m} \cdot 0.497 \frac{\mathrm{m}}{\mathrm{s}} \cdot 1.19 \frac{\mathrm{kg}}{\mathrm{m}^3}}{1.8 \cdot 10^{-5} \mathrm{Pas}} = 65.7$$

Solution 1: based on chart $\mathrm{Re}_m - f_m \cdot \mathrm{Re}_m^2$

Calculate $f_m \cdot \mathrm{Re}_m^2$ valid at balanced forces

$$f_m \cdot \mathrm{Re}_m^2 = \frac{d_p^3 \cdot (\rho_p - \rho) \cdot \rho \cdot g}{2 \cdot \eta^2} = \frac{(2 \cdot 10^{-3}\mathrm{m})^3 \cdot \left(2400 \frac{\mathrm{kg}}{\mathrm{m}^3} - 1.19 \frac{\mathrm{kg}}{\mathrm{m}^3}\right) \cdot 1.19 \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot (1.8 \cdot 10^{-5} \mathrm{Pas})^2}$$

$$f_m \cdot \mathrm{Re}_m^2 = 3.46 \cdot 10^5$$

According to the chart, the packing is in rest (because $\mathrm{Re}_m^* < \mathrm{Re}_m^* = 150$)

Use curve ε = 0.4 to read $f_m \cdot \mathrm{Re}_m^2$ belonging to Re_m = 65.7 .

$$f_m \cdot \mathrm{Re}_m^2 = 1.1 \cdot 10^5$$

$$f_m = \frac{(f_m \cdot \mathrm{Re}_m^2)}{\mathrm{Re}_m^2} = \frac{1.1 \cdot 10^5}{(65.7)^2} = 25.48$$

$$\Delta p = 4 \cdot f_m \cdot \frac{L_0}{d_p} \cdot \frac{v_0^2 \cdot \rho}{2} = 4 \cdot f_m \cdot \frac{L \cdot (1 - \varepsilon)}{d_p} \cdot \frac{v_0^2 \cdot \rho}{2}$$

$$\Delta p = 4 \cdot 25.48 \cdot \frac{2.4\mathrm{m} \cdot (1 - 0.4)}{2 \cdot 10^{-3}\mathrm{m}} \cdot \frac{\left(0.497 \frac{\mathrm{m}}{\mathrm{s}}\right)^2 \cdot 1.19 \frac{\mathrm{kg}}{\mathrm{m}^3}}{2} = 1.08 \cdot 10^4 \mathrm{Pa}$$

Solution 2: based on the relation of Δp_E and Δp_{grid}

$$\Delta p_E = \frac{1-\varepsilon}{\varepsilon^3} \cdot \frac{L}{d_p} \cdot \left[1.75 + \frac{150 \cdot (1-\varepsilon)}{Re_m}\right] \cdot v_0^2 \cdot \rho$$

$$\Delta p_E = .\frac{1-0.4}{0,4^3} \cdot \frac{2,4m}{2 \cdot 10^{-3} m} \cdot \left[1.75 + \frac{150 \cdot (1-0.4)}{65.7}\right] \cdot \left(0.497 \frac{m}{s}\right)^2 \cdot 1.19 \frac{kg}{m^3} = 1.03 \cdot 10^4 Pa$$

$$\Delta p_{grid} = L \cdot (1-\varepsilon) \cdot (\rho_p - \rho) \cdot g = 2.4m \cdot (1-0.4) \cdot \left(2400 \frac{kg}{m^3} - 1.19 \frac{kg}{m^3}\right) \cdot 9.81 \frac{m}{s^2} = 3.39 \cdot 10^4 Pa$$

$\Delta p_E < \Delta p_{rács}$, the packing is in rest, therefore

$$\Delta p = \Delta p_E = 1.03 \cdot 10^4 Pa$$

Problem 2

Liquid phase catalytic reaction is performed in a packed column of diameter 80 mm. The catalyst is carried on 3 mm diameter spherical particles of density 2500 kg/m^3. The liquid (ρ = 1200 kg/m^3, $\eta = 1.2\cdot10^{-3}$ Pas) is driven bottom up. Total packing weight is 8.5 kg.

Calculate:

a) Initial fluidization velocity (ε = 0.4)
b) Carry-out velocity (ε = 1)
c) Friction loss (pressure drop) over the packing if the liquid velocity is 20 % of the carry-out velocity
d) Packing height if the liquid velocity is 5 times larger than the initial fluidization velocity
e) Pressure drop over the packing at the velocity of d) if the packing is fixed in place from above

Solution

a) Initial fluidization velocity (ε = 0.4)

Calculate $f_m\cdot Re_m^2$ valid at balanced forces

$$f_m \cdot Re_m^2 = \frac{d_p^3 \cdot (\rho_p - \rho)\cdot \rho \cdot g}{2\cdot \eta^2} = \frac{(3\cdot 10^{-3}\,m)^3 \cdot \left(2500\frac{kg}{m^3} - 1200\frac{kg}{m^3}\right)\cdot 1200\frac{kg}{m^3}\cdot 9.81\frac{m}{s^2}}{2\cdot (1.2\cdot 10^{-3}\,Pas)^2}$$

$$f_m \cdot Re_m^2 = 1.43\cdot 10^5$$

At $f_m \cdot Re_m^2 = 1.43\cdot 10^5$ and ε = 0.4:

$$Re_m^* = 80$$

$$Re_m = \frac{d_p \cdot u_0 \cdot \rho}{\eta}$$

$$u_0^* = \frac{Re_m^* \cdot \eta}{d_p \cdot \rho} = \frac{80\cdot 1.2\cdot 10^{-3}\,Pas}{3\cdot 10^{-3}\,m\cdot 1200\frac{kg}{m^3}} = 2.67\cdot 10^{-2}\,\frac{m}{s}$$

b) Carry-out velocity (ε = 1)

At $f_m \cdot Re_m^2 = 1.43\cdot 10^5$ and ε = 1:

$$Re_m^{**} = 800$$

$$u_0^{**} = \frac{Re_m^{**} \cdot \eta}{d_p \cdot \rho} = \frac{800\cdot 1.2\cdot 10^{-3}\,Pas}{3\cdot 10^{-3}\,m\cdot 1200\frac{kg}{m^3}} = 0.267\,\frac{m}{s}$$

c) Friction loss (pressure drop) over the packing if the liquid velocity is 20 % of the carry-out velocity

Actual velocity:

$$u_0 = 0.2 \cdot u_0^{**} = 0.2 \cdot 0.267 \frac{m}{s} = 5.34 \cdot 10^{-2} \frac{m}{s}$$

$u_0 = 5.34 \cdot 10^{-2} \frac{m}{s} > 2.67 \cdot 10^{-2} \frac{m}{s} = u_0^{*}$, thus the packing is fluidized. Therefore the pressure drop equals the grid pressure.

Reduced packing height is needed for calculating the grid pressure. This can be obtained from the mass of the packing.

Net volume of the packing:

$$V_{packing} = \frac{m_{packing}}{\rho_p} = \frac{8.5 kg}{2500 \frac{kg}{m^3}} = 3.4 \cdot 10^{-3} m^3$$

Reduced packing height:

$$V_{packing} = L_0 \cdot A_{tube} = L_0 \cdot \frac{D_{tube}^2 \cdot \pi}{4}$$

$$L_0 = \frac{V_{packing}}{A_{tube}} = \frac{V_{packing}}{\frac{D_{tube}^2 \cdot \pi}{4}} = \frac{3.4 \cdot 10^{-3} m^3}{\frac{(0.08 m)^2 \cdot \pi}{4}} = 0.676 m$$

Grid pressure:

$$\Delta p = \Delta p_{grid} = L \cdot (1 - \varepsilon) \cdot (\rho_p - \rho) \cdot g = L_0 \cdot (\rho_p - \rho) \cdot g$$

$$\Delta p = 0.676 m \cdot \left(2500 \frac{kg}{m^3} - 1200 \frac{kg}{m^3} \right) \cdot 9.81 \frac{m}{s^2} = 8.6 \cdot 10^3 Pa$$

d) Packing height if the liquid velocity is 5 times larger than the initial fluidization velocity

Actual ε is to be determined.

Actual velocity:

$$u_0 = 5 \cdot u_0^{*} = 5 \cdot 2.67 \cdot 10^{-2} \frac{m}{s} = 0.1335 \frac{m}{s}$$

$$Re_m = \frac{d_p \cdot u_0 \cdot \rho}{\eta} = \frac{3 \cdot 10^{-3} m \cdot 0.1335 \frac{m}{s} \cdot 1200 \frac{kg}{m^3}}{1.2 \cdot 10^{-3} Pas} = 400$$

At $f_m \cdot Re_m^2 = 1.43 \cdot 10^5$ and Re_m = 400: ε = 0.8.

Packing height from reduced packing height:

$$L_0 = L \cdot (1 - \varepsilon)$$

$$L = \frac{L_0}{(1 - \varepsilon)} = \frac{0.676 m}{(1 - 0.8)} = 3.38 m$$

e) Pressure drop over the packing at the velocity of d) if the packing is fixed in place from above

No fluidization in this case, and $\varepsilon = 0.4$.

Actual packing height:

$$L = \frac{L_0}{(1-\varepsilon)} = \frac{0,676\text{m}}{(1-0.4)} = 1.127\text{m}$$

Ergun-formula:

$$\Delta p_E = \frac{1-\varepsilon}{\varepsilon^3} \cdot \frac{L}{d_p} \cdot \left[1.75 + \frac{150 \cdot (1-\varepsilon)}{Re_m}\right] \cdot v_0^2 \cdot \rho$$

$$\Delta p_E = \frac{1-0.4}{0.4^3} \cdot \frac{1.127\text{m}}{3 \cdot 10^{-3}\text{m}} \cdot \left[1.75 + \frac{150 \cdot (1-0.4)}{400}\right] \cdot \left(0.1335\frac{\text{m}}{\text{s}}\right)^2 \cdot 1200\frac{\text{kg}}{\text{m}^3} = 1.48 \cdot 10^5\,\text{Pa}$$

Problem 3

Polimer beads are dried with air, in a fluidizing dryer.

Data:

d_p = 2 mm

ρ_p = 1150 kg/m^3 ρ = 1.061 kg/m^3

ε = 0.4 $\eta = 2\cdot 10^{-5}$ Pas

Quantities to be determined:

a) Initial fluidization velocity
b) Grid pressure at reduced packing height 2 m

Solution

a) Initial fluidization velocity

Calculate $f_m \cdot Re_m^2$ at balanced forces

$$f_m \cdot Re_m^2 = \frac{d_p^3 \cdot (\rho_p - \rho)\cdot \rho \cdot g}{2\cdot \eta^2} = \frac{\left(2\cdot 10^{-3}\,\mathrm{m}\right)^3 \cdot \left(1150\frac{\mathrm{kg}}{\mathrm{m}^3} - 1.061\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\cdot 1.061\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot 9.81\frac{\mathrm{m}}{\mathrm{s}^2}}{2\cdot\left(2\cdot 10^{-5}\,\mathrm{Pas}\right)^2}$$

$$f_m \cdot Re_m^2 = 1.2\cdot 10^5$$

At $f_m \cdot Re_m^2 = 1.2\cdot 10^5$ and ε = 0.4:

$$Re_m^* = 70$$

$$Re_m^* = \frac{d_p \cdot u_0^* \cdot \rho}{\eta}$$

$$u_0^* = \frac{Re_m^* \cdot \eta}{d_p \cdot \rho} = \frac{70\cdot 2\cdot 10^{-5}\,\mathrm{Pas}}{2\cdot 10^{-3}\,\mathrm{m}\cdot 1.061\frac{\mathrm{kg}}{\mathrm{m}^3}} = 0.66\frac{\mathrm{m}}{\mathrm{s}}$$

b) Grid pressure at reduced packing height 2 m

L_0 = 2 m

$$\Delta p_{grid} = L\cdot(1-\varepsilon)\cdot(\rho_p - \rho)\cdot g = L_0 \cdot (\rho_p - \rho)\cdot g$$

$$\Delta p_{grid} = 2\mathrm{m}\cdot\left(1150\frac{\mathrm{kg}}{\mathrm{m}^3} - 1.061\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\cdot 9.81\frac{\mathrm{m}}{\mathrm{s}^2} = 2.25\cdot 10^4\,\mathrm{Pa}$$