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Chapter 1

Fluid flow

1.1 Incompressible and frictionless fluids

Liquids are almost incompressible. Even by applying large compressing force, the density increase of the liquid is usually negligible. (The energy needed to compress the liquid is, however, considerable.)

Gases are usually considerably compressible. However, low pressure gases may be considered approximately incompressible if they flow through a channel and the pressure difference between the two end points of the channel is small. For example, if a blower drives air through a packed pipe with 1.2 bar at the inlet point of the pipe, and atmospheric pressure at the outlet, then the air can be approximately modelled as incompressible fluid.

The **Law of continuity** applies to incompressible fluids. Should the fluid flow through a channel of cross section area A with **volumetric flow rate** W , then its (average) speed u is expressed as

$$u = \frac{W}{A}$$

where A may be measured e.g. in m^2 , W in m^3/s , and u in m/s . This is valid even if the channel is wider or narrower at some places. Thus, if the cross section areas are A_1 , A_2 , and A_3 at different places of the channel, then the speeds are, respectively,

$$u_1 = \frac{W}{A_1}, \quad u_2 = \frac{W}{A_2}, \quad \text{and} \quad u_3 = \frac{W}{A_3}$$

The same relation can be expressed as the constancy of the flow rate in the channel irrespectively to the actual cross section and the speed:

$$W = u_1 A_1 = u_2 A_2 = u_3 A_3$$

The Law of continuity is valid irrespective to the presence or lack of friction, but incompressibility and conservation of material are assumed. Should a leakage alongway the channel be present so that the fluid can escape, the continuity is no more valid.

Fluid friction can be neglected for practical purposes in some cases. These include those cases where the friction forces are small compared to the other forces acting on the fluid.

When neither friction nor compression is taken into account, and when the gravitational acceleration is also considered constant, then the energy conservation related to the fluid in motion can be expressed as

$$\frac{\rho V u^2}{2} + \rho V g h + p V = \text{constant}$$

where V is a very small volume of the fluid, ρ is its (constant) density, g is gravitational acceleration (appr. 9.81 m/s^2), p is pressure, and h is height, measured from an arbitrary level, of the point where volume V is situated. Here it is assumed that no energy source or sink, acting the the moving fluid is present. The first member is the *kinetic energy* of the fluid, the second one is its *potential energy* in the gravitational field, and the third one expresses the energy needed to reach the actual pressure from a state of zero pressure. (You can check that all the members have energy dimension.)

Selection of the small V volume is arbitrary, and the equation can be divided by it, giving the famous **Bernoulli equation**:

$$\boxed{\frac{\rho u^2}{2} + \rho g h + p = \text{constant}}$$

This is the so-called *pressure form* of the Bernoulli equation; here all the members are expressed in pressure dimension. The first member is the kinetic pressure, the second one is the hydrostatic pressure, whereas the last one is the internal pressure of the fluid.

By dividing the equation with ρ , the members are expressed in specific energy, i.e. in energy related to unit mass:

$$\frac{u^2}{2} + g h + \frac{p}{\rho} = \text{constant}$$

Now, if this equation is divided by g , then all the members are expressed in height, also called *head* in engineering practice:

$$\frac{u^2}{2g} + h + \frac{p}{\rho g} = \text{constant}$$

Here the first member is the so-called kinetic head, the second one is the geodetic head, and the third one is the pressure head.

Considering two equal V volumes of the fluid, at different points, then the pressures, speeds, and heights can be different, but the energy should remain the same. Thus

$$\frac{u_1^2}{2g} + h_1 + \frac{p_1}{\rho g} = \frac{u_2^2}{2g} + h_2 + \frac{p_2}{\rho g}$$

and the other two forms can also be applied in the same way.

The head form of the equation is useful in solving problems considering pumping and fluid flow in pipes, whereas the other two forms are useful in computing gas compressors.

Measuring flow rate with continuity. Volumetric flow rate can be measured by pressure difference developing between two different cross sections of the same channel. *Venturi tube*, *flow nozzle*, and *orifice* are such devices. Considering horizontal arrangement, or negligible height difference between the two points of the pressure measurement (**figure *****), the pressure form of Bernoulli equation is reduced to

$$\frac{u_1^2}{2g} + \frac{p_1}{\rho g} = \frac{u_2^2}{2g} + \frac{p_2}{\rho g}$$

so that the measured pressure drop is

$$\Delta p \approx p_1 - p_2 = \frac{\rho(u_2^2 - u_1^2)}{2}$$

This equality is just an approximating one because neither fluid friction nor contraction effects are taken into account.

Liquid outflow through a small hole. Consider a wide vessel filled with liquid up to level h , with a small hole of cross section negligible compared to the cross section of the vessel (**figure *****). The head form of the Bernoulli equation is now

$$\frac{u_{\text{level}}^2}{2g} + h + \frac{p_{\text{above}}}{\rho g} = \frac{u^2}{2g} + 0 + \frac{p_{\text{below}}}{\rho g}$$

The liquid loss of the vessel is negligible, thus $u_{\text{level}} \approx 0$ may be taken. If atmospheric (or, generally, equal) pressure is present both above and below the vessel, the equation is reduced to

$$h \approx \frac{u^2}{2g}$$

i.e.

$$u \approx \sqrt{2gh}$$

If the radius of the hole is r then the volumetric flow rate W is approximately $W \approx r^2\pi\sqrt{2gh}$. This formula is due to *Toricelli*, the Italian physicist who first measured atmospheric pressure. However, the actual flow is contracted more than

the hole's cross section. This apparently higher contraction is taken into account by a so-called *contraction factor* $\alpha < 1$: $u_2(\alpha A_2) = u_1 A_1$. Thus,

$$W \approx \alpha r^2 \pi \sqrt{2gh}$$

A similar effect occurring in head flow meters such as orifice, *Venturi-tube*, and flow nozzle, is taken into account by a so-called *flow coefficient*.

1.2 Fluid friction

When a plug with outer shape fitting to the inner shape of a channel moves inside the channel, the inner wall of the channel expresses a force against the movement of the plug. This phenomenon is called **friction**. When fluid flows in the pipe then the friction expressed by the channel wall to the fluid particles near the wall is mediated by the fluid to the particles farther from the wall. However, because of the elasticity of the fluid, those particles farther from the wall move faster than those drawn back by the friction of the wall. On the other hand, the particles nearer the wall express a force against the faster movement of the particles next to them. This is also a kind of friction, called **internal friction of the fluid**. This phenomenon occurs when layers of the fluid move in different speed.

Bernoulli equation with friction. If friction cannot be neglected then the energy loss due to friction is to be taken into account. The 'energy loss' is counted on the hitherto considered forms of energy: kinetic, potential, and pressure. No energy is lost in reality but these forms of energy are transformed to heat, i.e. internal energy of the fluid. In case of isotherm systems, this heat energy is dissipated to the environment through the wall. Thus, the energy balance (*Bernoulli equation*) is

$$\frac{\rho u_1^2}{2} + \rho g h_1 + p_1 = \frac{\rho u_2^2}{2} + \rho g h_2 + p_2 + \Delta p_f$$

or

$$\frac{u_1^2}{2g} + h_1 + \frac{p_1}{\rho g} = \frac{u_2^2}{2g} + h_2 + \frac{p_2}{\rho g} + h_f$$

where index f refers to friction. Δp_f is pressure drop due to friction; h_f is head loss due to friction.

Newtonian fluids. The force F of friction is proportional to the surface or area A at which the moving bodies or layers touch. When two such fluid layers slip on each other (**figure *****) then a unit force called *shear stress* is formed. (Stress and pressure have the same dimension.) Newtonian fluid is a simplest model describing

this stress. A fluid is Newtonian if the shear stress is proportional to the differential of speed perpendicular to the surface:

$$\frac{F_z}{A_{z,y}} = -\eta \frac{du_z}{dx}$$

Here z , y , and x are perpendicular directions, u_z is the speed component along direction z , F_z is the force acting in direction z , and $A_{z,y}$ is a planar surface perpendicular to direction x . η is called dynamic viscosity, and is independent of direction in case of a Newtonian fluid. On the other hand, η is a property specific to the actual material, and depends on temperature and pressure.

Non-Newtonian fluids. Quite many fluids may be considered Newtonian, or may be approximately modelled this way. However, there are fluids behaving in different ways (**figure *****).

Bingham-plastic fluids can be described by equation

$$\frac{F}{A} = \left(\frac{F}{A}\right)_0 - B \frac{du_z}{dx}$$

Such fluids are dense suspensions, pastes, sludges. B is called apparent (or plastic) viscosity, $(F/A)_0$ is called yield limit.

Pseudoplastic fluids have concave stress curve against speed slope, and can be approximated by the *Ostwald-de-Waele* model:

$$\frac{F}{A} = -B' \left(\frac{du_z}{dx}\right)^n$$

($n < 1$). The slope of the curve decreases as the speed slope increases. Such materials are e.g. polymeric solutions, melts, dyes, etc.

Dilating fluids have convex curve, with the same model but $n > 1$. (At $n = 1$ the Newtonian fluid is modelled.) Such fluids are dense suspensions like wet sand, dense dust in water.

There are other classes like *tixotrop* fluids whose apparent viscosity depends on the time of the shearing effect as well, or the *Maxwellian* fluids which are flexible like plastics, bitumens, pastas, gums.

1.3 Laminar flow of Newtonian fluid

Consider flow of a Newtonian fluid in a straight pipe with circular cross section of internal diameter D , radius $R = D/2$. Experience shows that the fluid moves faster in the centerline of the pipe than near the wall. If the flow is not too fast (what this 'too' means turns out in the next section) then one may suppose that circular layers of the fluid slip over each other, as is shown in **figure *****. We speak about **laminar flow** or **streamline flow pattern** in this case. The fluid is

pressed through the pipe by the pressure difference $p_1 - p_2$ between the ends of the pipe of length L . Any central rod of radius r is pressed by the force acting on its cross section $r^2\pi$, whereas the shearing stress measured at its outer circular surface acts on its superficies $2r\pi L$. The fluid moves with constant average speed u and constant local speed $v(r)$ in direction z (the axis) if the sum of the two forces is zero:

$$\Delta p r^2 \pi + \eta \frac{dv}{dr} 2\pi r L = 0$$

Thus

$$\frac{dv}{dr} = -\frac{\Delta p}{2\eta L} r$$

and

$$\int_0^{v_{max}} dv = -\frac{\Delta p}{2\eta L} \int_R^r dr$$

which then gives the axial speed profile as

$$v = \frac{\Delta p}{4\eta L} (R^2 - r^2)$$

From here

$$v_{max} = \frac{\Delta p}{4\eta L}$$

at $r = 0$ (the axis), and

$$v(r) = v_{max} \left(1 - \frac{r^2}{R^2}\right)$$

The volume flowing through (per time) the cylindric layer at radius r and width dr is $v(r)2r\pi dr$, thus total flow rate is

$$W = \int_0^R v(r)2r\pi dr = \int_0^R \frac{\Delta p}{4\eta L} (R^2 - r^2) 2r\pi dr$$

Integration gives the well-known **Hagen–Poiseuille equation**:

$$W = \frac{\Delta p \pi}{8\eta L} R^4$$

From here, the average speed u is

$$u = \frac{\Delta p}{8\eta L} R^2 = \frac{v_{max}}{2}$$

The local speed is symmetric to the axis, and of parabolic shape, shown in **figure *****. Note that the pressure difference Δp needed to drive the flow is proportional to the speed u :

$$\Delta p = \frac{8\eta L}{R^2} u = \frac{32\eta L}{D^2} u$$

1.4 Turbulent flow

The pressure difference Δp needed to drive the flow is proportional to the speed u if u is small, but becomes quadratic in u at higher speed, according to experience. The general shape of this dependency for Newtonian fluid is shown in **figure *****. The function has branches according to roughness of the pipe wall. This quadratic behaviour is due to the so-called **turbulent flow pattern** forming at higher speed.

The so-called *Reynolds* experiment is shown in **figure *****. A narrow thread of ink is slightly widened by diffusion as it is carried away by a slow stream of water in the glass pipe of diameter D . The laminar (streamline) flow is validated. By gradually increasing the flow rate of the carrying stream, the pattern does not change until a certain flow rate where it suddenly and radically changes. The thread of the ink is broken so much that the whole cross section becomes coloured right at the point of entering the pipe. There is a strong turbulence in the flow. In this case the speed profile can be approximated by

$$v(r) = v_{max} \left(1 - \frac{r}{R}\right)^{\frac{1}{7}}$$

(see **figure *****).

By experiments, for Newtonian fluids, laminar flow pattern is prevalent under

$$u < 2300 \frac{\eta}{D\rho} \equiv 2300 \frac{\nu}{D}$$

where ν is kinetic viscosity. Thus, a dimensionless number, called **Reynolds number** can be formed as

$$Re = \frac{Du\rho}{\eta} = \frac{Du}{\nu}$$

so that at $Re < 2300$ the flow is laminar. Above this critical Reynolds number, the flow pattern may be turbulent, and the higher speed involves higher chance to have turbulency. Above 4000, the flow is most probable turbulent, although laminar flow has been observed even at $Re > 10000$. The three-dimensional differential equations describing the flow of the fluid have alternative solutions, and thermodynamic stability criterions can be applied. However, each solution is stable in a small neighbourhood, and whether laminar flow is maintained depends on how much random disturbance is applied on the system.

Friction can be modelled as follows. The driving force needed to overcome the friction is:

$$F_{driv} = \Delta p_f \frac{D^2 \pi}{4}$$

The friction force is proportional to the rubbed surface $D\pi L$ and the specific kinetic energy $\frac{1}{2}\rho u^2$:

$$F_{fric} = 4f (D\pi L) \frac{\rho u^2}{2}$$

Here f is a so-called **friction factor**, and the multiplier 4 is arbitrarily introduced. In steady flow the two forces must be balanced (equal) so that

$$\Delta p_f = f \frac{L}{D} \frac{\rho u^2}{2}$$

This friction factor definition is used by *Blasius* and *Darcy*. The friction factor has earlier been defined by *Fanning* as

$$\Delta p_f = 4f \frac{L}{D} \frac{\rho u^2}{2}$$

because in that case f expresses a ratio of shear stress to the kinetic energy. One has to be careful to check which definition is applied in the actual equation or plot found in the reference books or articles.

Friction factor f is used to describe how friction depends on speed. In practice, however, friction depends on the roughness of the wall, as well. At low Re numbers (laminar flow) f can be expressed using the *Hagen-Poiseuille* equation. Comparing the equations, one gets

$$f_{\text{laminar}} = \frac{64}{Re}$$

At turbulent flow, dependence of f on Re and roughness is determined by experiments. Roughness is measured in length as unevenness or wrinkle k of the wall, and is usually related to the diameter of the pipe. Thus, f is expressed as a function of Reynolds number Re and relative roughness k/D . The *Moody* plot, applying logarithmic scale, is usually applied in practice for Newtonian fluids (see **Fig. 1.1**). At well developed turbulence, $Re \geq 4000$, the plot may be approximated as

$$\frac{1}{\sqrt{f}} \approx -2 \lg \left(\frac{2.51}{Re \sqrt{f}} + \frac{1}{3.72} \frac{k}{D} \right)$$

This equation should be solved for f by iterative calculation. The first estimate is taken as

$$f_0 = 0.25 \left[\lg \left(\frac{5.74}{Re^{0.9}} + \frac{1}{3.72} \frac{k}{D} \right) \right]^{-2}$$

and is corrected step by step according to

$$f_{i+1} = \left[-2 \lg \left(\frac{2.51}{Re \sqrt{f_i}} + \frac{1}{3.72} \frac{k}{D} \right) \right]^{-2}$$

The curves approach a horizontal line at the limit of high Re numbers; here f does not depend on Re , and thus Δp becomes exactly quadratic in u . This region can be approximated as

$$\frac{1}{\sqrt{f_{\text{rough}}}} \approx -2 \lg \left(\frac{1}{3.72} \frac{k}{D} \right)$$

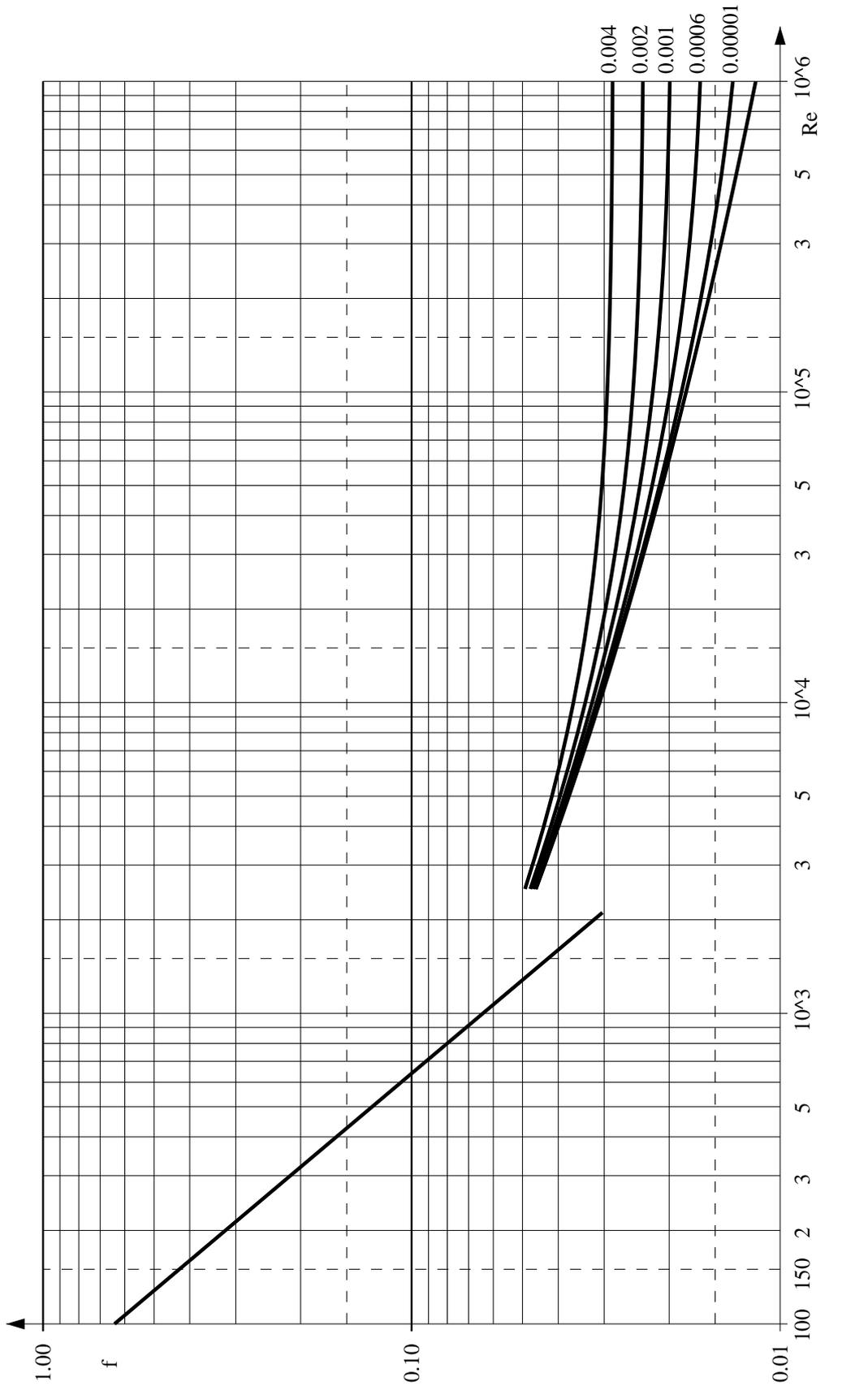


Figure 1.1: Friction factor in straight circular pipes

g replacements
 f
 Re
 10^3 0.10
 10^4
 10^5
 10^6
 100
 150
 2
 3
 5
 1.0
 0.1
 0.01
 0.004
 0.002
 0.001 0.01
 0.0006
 0.0001
 0.0001

This is approximately valid if

$$Re > 200 \frac{D}{k} \frac{1}{\sqrt{f}}$$

The branches of roughness are limited from below by a line of perfect smoothness. However, no perfectly smooth wall exists and the pressure drop approaches quadratic dependence on u at higher Re numbers in all practical cases.

This line of perfect smoothness was approximated by *Blasius* as

$$f_{smooth} \approx \frac{0.3164}{Re^{0.25}}$$

However, this is a straight line in the logarithmic plot, whereas the original line is a curve.

Moody plots are different for flow along planar walls.

For pipes with cross section not circular, equivalent diameter D_e , instead of D , may be substituted in the equations. Equivalent diameter is defined as

$$D_e = 4 \frac{A}{C}$$

where A is the cross section area and C is the wetted circumference. For circular pipes $D_e = D$.

For example, a rectangular cross section with sides a and b has cross section area is $A = ab$, circumference is $C = 2(a + b)$, thus the equivalent diameter is

$$D_e = \frac{2ab}{a + b}$$

Another example is the annular channel. This occurs when the fluid flows in the outer part of a coaxial double pipe (see **figure *****). Let the two radii be $R_1 < R_2$, then the cross sectional area is $A = (R_2^2 - R_1^2) \pi$, the wetted circumference is $C = 2(R_2 + R_1) \pi$, thus the equivalent diameter is

$$D_e = \frac{2(R_2^2 - R_1^2)}{(R_2 + R_1)} = 2(R_2 - R_1)$$

1.5 Hydraulic similarity and dimensional analysis

Newton's law of motion (force equals mass times acceleration) can be applied to moving fluids as well. Instead of considering the general case when acceleration is present, here we deal with the stationary case only, i.e. when the fluid moves with constant speed. No acceleration occurs when the algebraic sum of all the forces acting on the fluid is zero. The forces to be counted are inertia of the moving fluid,

the pressing force, the counteracting friction force, and gravity if present in the direction of the flow. An energy term can be attached to each of these forces. The energy terms related to unit volume can be formulated by variables as

$$\frac{1}{2}\rho u^2, \quad \Delta p, \quad \eta \frac{du_x}{dz}, \quad \rho gh$$

For a general case, $\frac{du_x}{dz}$ can be considered by a ratio of u to a characteristic length D : $\frac{u}{D}$. Two flows (in two different arrangements) can be considered similar (or congruent, or isomorph) if the ratios of these members are equal. Thus, dimensionless criteria are formed by relating one member to the other. The most frequently applied similarity criteria are:

Reynolds number: $Re = \frac{\text{inertia}}{\text{friction}} = \frac{Du\rho}{\eta} = \frac{Du}{\nu}$
--

Euler number: $Eu = \frac{\text{pressure}}{\text{inertia}} = \frac{p}{\rho u^2}$

Froude number: $Fr = \frac{\text{inertia}}{\text{gravity}} = \frac{u^2}{gh}$

Models related to flow should be consistent in dimensions. When a relation between the variables is expressed explicitly to zero ($0 = f(x, y, z, \dots)$), the function should give a dimensionless value. Assuming a polynomial form with variables $\Delta p, u, D, \eta, \rho, g$, and some other characteristic length L , i.e.

$$\Pi = (\Delta p)^a u^b D^c \eta^d \rho^e g^f L^h$$

where Π is a dimensionless expression, the following equation should be valid

$$1 = \left(\frac{M}{LT^2}\right)^a \left(\frac{L}{T}\right)^b L^c \left(\frac{M}{LT}\right)^d \left(\frac{M}{L^3}\right)^e \left(\frac{L}{T^2}\right)^f L^h$$

where L, M, and T stand for dimensions of Length, Mass, and Time, respectively. Thus, the following linear equations must be satisfied by the exponents:

$$\begin{aligned} \text{for T: } & 0 = -2a - b - d - 2f \\ \text{for M: } & 0 = a + d + e \\ \text{for L: } & 0 = -a + b + c - d - 3e + f + h \end{aligned}$$

b and e are expressed from T and M:

$$\begin{aligned} b &= -2a - d - 2f \\ e &= -a - d \end{aligned}$$

and substituted to L, and the c is expressed from L:

$$c = -d + f - g$$

so that the b, e, and c can be discarded:

$$\Pi = (\Delta p)^a u^{-2a-d-2f} D^{-d+f-h} \eta^d \rho^{-a-d} g^f L^h$$

Now the constituents can be rearranged according to single common exponents:

$$\Pi = \left(\frac{\Delta p}{u^2 \rho} \right)^a \left(\frac{\eta}{u D \rho} \right)^d \left(\frac{D}{u^2 g} \right)^f \left(\frac{L}{D} \right)^h$$

i.e.

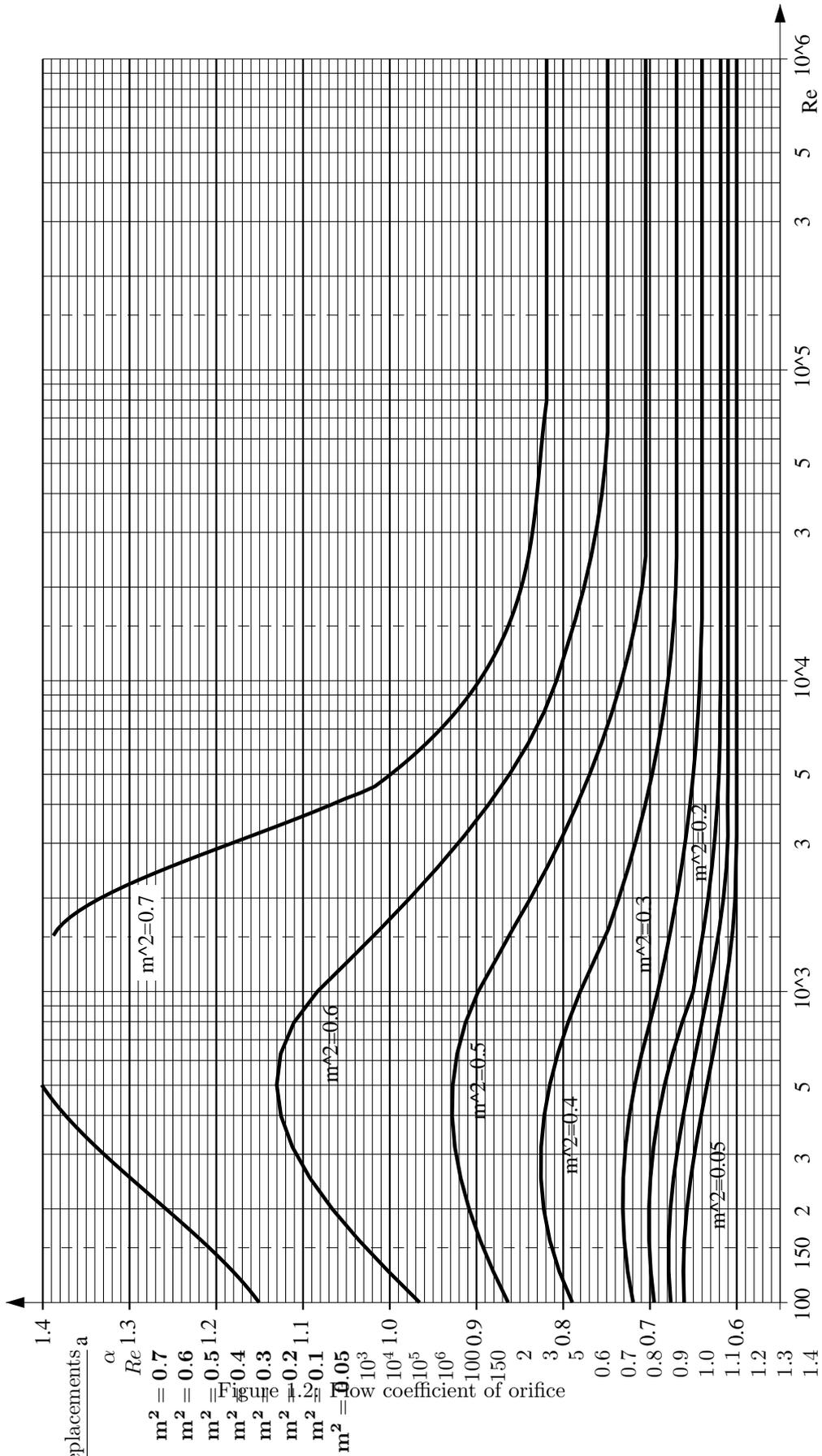
$$\Pi = Eu^a Re^{-a} Fr^{-f} \left(\frac{L}{D} \right)^h$$

Thus, any of the dimensionless numbers can be expressed as a polynomial of the others and some geometric ratios. The exponents can then be determined by fitting them to experimental data. Such a dimensional analysis followed by data fitting can be utilized to obtain information characteristic to geometrically similar arrangements. Once such a characteristic relation is measured and fitted, it can be used to predict the behaviour of other systems that are similar to the measured one geometrically. Any change in materials properties (viscosity, density) and speed is absorbed in a shift in the actual values of the dimensionless numbers, but the relation remains unchanged. The *Moody* plot shows just one such a general relation.

Another example is the flow coefficient α of a given geometry orifice as function of Re . The flow coefficient is tabulated in reference books, and depends on the geometry. Such a dependence is shown in **Fig. 1.2**. Here $m = \frac{d}{D}$ where D is the pipe's internal diameter, and d is the orifice diameter. The volumetric flow rate can be calculated as

$$W = \alpha \varepsilon \frac{d^2 \pi}{4} \sqrt{\frac{2 \Delta p}{\rho}}$$

where ε is volumetric contraction factor, $\varepsilon = 1$ if the fluid is incompressible. This particular plot is valid if the taps are made before the orifice with D and after the orifice with $\frac{1}{2}D$.



Chapter 2

Sedimentation (settling)

2.1 Drag coefficient

The fluid collides with the body over a cross section area A_{coll} of collision and rubs or chafes the the passed surface A_{fric} . The force exerted by the collision can be modelled as

$$F_{coll} = A_{coll} \frac{u^2 \rho}{2}$$

whereas the friction force at the rubbed surface can be modelled as

$$F_{fric} = f A_{fric} \frac{u^2 \rho}{2}$$

The total force is called **drag force** because the fluid drags the body:

$$F_{drag} = F_{coll} + F_{fric}$$

However, measuring (and even defining) the rubbed surface is difficult in practice. Thus, the collision surface is first used instead of the passed one, and a modified friction factor f' is applied as

$$F_{fric} = f A_{fric} \frac{u^2 \rho}{2} = f' A_{coll} \frac{u^2 \rho}{2}$$

so that the two members can be added

$$F_{drag} = (1 + f') A_{coll} \frac{u^2 \rho}{2}$$

Finally, a so-called **drag coefficient** C_D is defied as

$$F_{drag} = C_D A_{coll} \frac{u^2 \rho}{2}$$

C_D can be experimentally determined as a function of **settling Reynolds number**

$$Re = \frac{du\rho}{\eta}$$

where d is a characteristic length of the body. The actual shape of this function depends of the shape of the body. **Figure ***** shows this function for spherical particles (solid balls) of diameter d , and for straight rods of circular cross section with diameter d , in case when the fluid flow is perpendicular to the rod.

Stokes region. *Stokes* determined, in 1851, the drag force exerted on a ball by fluid of dynamic viscosity η and moving with low speed u as

$$F_{drag} = 3\pi\eta du$$

The same equation can be derived by taking the experimentally measured

$$C_D = \frac{24}{Re}$$

at low Reynolds numbers ($Re < 0.6$):

$$F_{drag} = \frac{24\eta}{du\rho} \frac{d^2\pi}{4} \frac{u^2\rho}{2} = 3\pi\eta du$$

In practice, the behaviour can be considered laminar up to $Re \leq 4$.

Note that in this laminar region F_{drag} is proportional with d and u , and does not depend on ρ .

Newton region. In the region of developed turbulency (at $800 < Re < 2 \cdot 10^5$), named after *Newton*, the drag coefficient is independent of the Reynolds number, and can be taken as $C_D \approx 0.44$. Thus,

$$F_{drag} \approx 0.44 \frac{d^2\pi}{4} \frac{u^2\rho}{2} \approx 0.1726d^2u^2\rho$$

Note that in this turbulent region F_{drag} is proportional with d^2 , u^2 , and ρ .

Transient region. In between laminar flow and developed turbulency the drag coefficient may be approximated as

$$C_D \approx \frac{18.5}{Re^{0.6}} \approx \frac{12}{\sqrt{Re}}$$

Limits of validity. At very high speed, sound velocity effects may influence the behaviour of the fluid. Validity of the Stokes region is limited to the scale where the fluid may be modelled as a continuum. Thus, the Stokes region is valid only if $d > L_{free}$ where L_{free} is the average free pathlength of the molecules constituting the fluid. Below this scale, the particles behave according to the Brownian motion. At even lower scale, commensurable with the molecule length, diffusion occurs instead of Brownian motion.

Shape factor. Behaviour of non-spherical particles can be approximated by the relations determined to spherical particles but applying so-called **shape factor** ϕ as a multiplier of C_D :

$$C_D = \phi C_{D,spherical}$$

2.2 Terminal velocity

Balance of forces. There are three forces acting on a particle moving in a standing fluid or on a standing particle around which the fluid flows. For brevity, we consider a heavy particle falling in a steady fluid. The gravity force (commonly called the weight) acts downwards $F_W = m_p g = V \rho_p g$ where index p refers to the particle, V is its volume, m is mass, ρ is density, and g is the gravitational constant. Buoyancy force acts against this weight, i.e. upwards and, according to *Archimedes*, is $F_B = V \rho_f g$ where index f refers to the fluid. The difference of these two forces is called the *Archimedian weight*: $F_A = V (\rho_p - \rho_f) g$. When the particle falls, however, there is a third force acting against its move: the drag force F_D . Acceleration of the particle downwards, according to *Newton* is:

$$\frac{dm_p u}{dt} = F_A - F_D$$

Whereas F_A is determined by the density difference and the volume of the particle, and is thus constant, the drag force counteracting the fall increases with the speed of falling. As a result, small particles very soon reach a speed at which the drag force just balances the Archimedian weight, and the speed stops increasing. This speed is called **terminal velocity of falling**. In this limit the forces are in equilibrium, i.e.

$$\frac{dm_p u}{dt} = V (\rho_p - \rho_f) g - C_D A_{coll} \frac{u^2 \rho_f}{2} = 0$$

The terminal velocity can be expressed from this balance.

Spherical particles. Since for spherical particle (little ball or globe)

$$A_{coll} = \frac{d^2 \pi}{4}; \quad V = \frac{d^3 \pi}{6}$$

the balance is

$$\frac{d^3\pi}{6}\Delta\rho g = C_D \frac{d^2\pi}{4} \frac{u^2\rho_f}{2}$$

and the speed can be expressed as

$$u^2 = \frac{1}{C_D} \frac{4}{3} \frac{\Delta\rho}{\rho_f} gd$$

$$u = \sqrt{\frac{1}{C_D} \frac{4}{3} \frac{\Delta\rho}{\rho_f} gd}$$

Stokes region. Here

$$C_D = \frac{Re}{24} = \frac{24\eta}{du\rho_f}$$

and thus

$$u^2 = \frac{du\rho_f}{24\eta} \frac{4}{3} \frac{\Delta\rho}{\rho_f} gd$$

from where u can be expressed:

$$u = \frac{\Delta\rho g}{18\eta} d^2$$

Note that u is proportional to d^2 and $\frac{\Delta\rho}{\eta}$.

Newton region. Here $C_D \approx 0.44$ and thus

$$u \approx 1.74 \sqrt{\frac{\Delta\rho}{\rho_f} gd}$$

Note that u is proportional to \sqrt{d} and $\sqrt{\frac{\Delta\rho}{\rho_f}}$.

Transient region. Here, applying the square root approximation,

$$u \approx \frac{\Delta\rho}{9\sqrt{\eta\rho_f}} gd$$

Note that u is approximately proportional to d .

2.3 Calculations

Given the material properties and the characteristic d , the expected terminal speed can be computed. Conversely, d can be computed from a measured u . Use of the $C_D \longleftrightarrow Re$ plot is, unfortunately, not straightforward because both u and d are needed to compute Re . Thus, in principle, iterative calculation is suggested.

Re	F_u	F_d
4	2.2	0.18
800	20	12
$2 \cdot 10^5$	2000	70

Table 2.1: Settling regions' criteria

For spherical particles, this difficulty can be avoided by expressing $Re^2 C_D$ and $\frac{Re}{C_D}$, or their cubic root. Namely

$$\left(\frac{Re}{C_D}\right)^{\frac{1}{3}} = \frac{u}{B\nu} \equiv F_u$$

$$(C_D Re^2)^{\frac{1}{3}} = Bd \equiv F_d$$

where

$$B = \frac{4}{3}g \frac{\rho_f \Delta \rho}{\eta^2}$$

Here B depends on the material properties only, F_u depends on B and u only, F_d depends on B and d only. Based on these new variables, a generalized settling chart can be constructed as shown in **figure *****. This figure is applicable to compute u from d or d from u without iteration.

Even this graphical procedure can be avoided and substituted by an approximation because F_u , Re , and F_d depend on each other monotonically increasing. Thus, the critical Re numbers can be projected to F_u and F_d as shown in **Table 2.1**. The explicite formulas given below are approximations only. In practice, the drag force is greater than that computed with the formula given to the laminar region if $Re > 1$. The formula given to the transient region assigns a straight line in the logarithmic plot, whereas the measured data follow a curve.

Stokes region:

$$F_u = \frac{F_d^2}{24} \quad F_d = \sqrt{24F_u} \quad \left(\begin{array}{l} Re = \sqrt{24F_u^3} \\ Re = \frac{F_d^3}{24} \end{array} \right)$$

Newton region:

$$F_u \approx \frac{\sqrt{F_d}}{0.44} \quad F_d \approx (0.44F_u)^2 \quad \left(\begin{array}{l} Re \approx 0.44F_u^3 \\ Re \approx \sqrt{\frac{F_d^3}{0.44}} \end{array} \right)$$

Transient region:

$$F_u \approx \sqrt[3]{\frac{F_d^2}{12^5}} \quad F_d \approx \sqrt{12^5 F_u^3} \quad \left(\begin{array}{l} Re \approx \sqrt[3]{144F_u^2} \\ Re \approx \frac{F_d^2}{\sqrt[3]{144}} \end{array} \right)$$

Co-settling. Which particles fall together? I.e., which particles fall with the same speed?

If the particles are of the same kind, i.e. each have the same density ($\rho_1 = \rho_2$), then the equal particle size ($d_1 = d_2$) fall together.

If the particles are different in density ($\rho_1 \neq \rho_2$) then different size particles ($d_1 \neq d_2$) fall together.

In *Stokes region* (laminar flow) $u_1 = u = u_2$ if

$$\frac{\rho_1 - \rho_f}{18\eta} g d_1^2 = \frac{\rho_2 - \rho_f}{18\eta} g d_2^2$$

i.e.

$$\frac{\rho_1 - \rho_f}{\rho_2 - \rho_f} = \frac{d_2^2}{d_1^2}$$

Thus, if $\rho \gg \rho_f$ then

$$\frac{\rho_1}{\rho_2} \approx \frac{d_2^2}{d_1^2}$$

In *Newton region* (turbulent flow) $u_1 = u = u_2$ if

$$\sqrt{\frac{\rho_1 - \rho_f}{\rho_1} d_1} = \sqrt{\frac{\rho_2 - \rho_f}{\rho_2} d_2}$$

i.e.

$$\frac{1 - \frac{\rho_f}{\rho_1}}{1 - \frac{\rho_f}{\rho_2}} = \frac{d_2}{d_1}$$

Settling in mass. Consider **figure ***** where e.g. sand is falling in water in a test tube. The settling is slower in this case than when a single particle falls in a wide space. This is so because the liquid pressed out of the tube is forced to flow upward, and thus exerting a drag force higher than if it were standing. Or, one can consider this situation as falling with a greater speed than that related to the wall of the tube. Really, each particle moves against the water with its speed along the wall plus the speed of the water upward.

The speed u_0 computed till now is valid only if the particle falls alone in an arbitrary wide cross section of fluid. This is expressed as **terminal velocity in infinite space**. This should be approximately valid for the speed relative to the fluid; thus one may write $u_0 = u + u_{counter}$ where u is the speed related to the wall, and $u_{counter}$ is the speed of the fluid counterflowing (moving upward). This simple picture is, however, modified by the flow pattern or *Re* number different from which would be experienced otherwise. There are some approximating empirical formulas for calculating the speed, like

$$u \approx \frac{u_0}{1 + 2.4 \frac{d}{D}} \quad \text{or} \quad u \approx u_0 \left[1 - \frac{d}{D} \right]^{2.25}$$

A special case of such a situation is the *Höppler* viscosimeter. This is a usually slant straight glass tube filled with liquid to be measured and a ball almost but not as wide as the internal channel. The ball rather slowly falls down in the tube, and the falling speed can be used to determine the viscosity of the fluid.

Another special case is suspension settling. Suspensions are characterized with a *suspension percent* s as

$$s = \frac{\text{solid volume}}{\text{total volume}} \cdot 100\%$$

Then the actual speed can be expressed as $u \approx \varphi u_0$ where this φ factor is plotted against s as in **figure *****.

Settling in centrifugal field. If the gravity is not strong enough to separate in an acceptable speed the solid from gas or liquid, or liquid drops from gas, or separate two liquid phases, then the driving force of settling can be increased by centrifuge.

Denote the rotation number [turns/s] by n , then the angle speed expressed in radians is

$$\omega = 2\pi \left[\frac{1}{\text{turns}} \right] \cdot n \left[\frac{\text{turns}}{\text{s}} \right] = 2\pi n \left[\frac{1}{\text{s}} \right]$$

The centrifugal acceleration is

$$g_c \equiv \omega^2 r = 4\pi^2 n^2 r \left[\frac{\text{m}}{\text{s}^2} \right]$$

For characterizing the centrifuge, one may apply a so-called *centrifuge index* Z :

$$Z \equiv \frac{g_c}{g} = \frac{\omega^2 r}{g} = \frac{4\pi^2}{g} n^2 r \sim n^2 r$$

where g is the gravitational acceleration ($\approx 9.81 \text{ m/s}^2$). One may say 'how many g -s are reached'.

In *Stokes region*: $u = Z u_0$; in *Newton region*: $u = \sqrt{Z} u_0$.

2.4 Capacity

Capacity of a settling tray. Consider a rectangular channel as is shown in **figure ***** with length L , width b , and height H . Its cross section area $A_{\text{cross}} = bH$; its base area is $A = Lb$. Imagine a tray below the base, with any depth. The gas flowing through the channel carries solid particles (e.g. dust in gas). A distribution of smaller and larger particles is usually carried; settling out a fraction of heavier particles is targeted. This fraction is characterized with their smallest terminal falling velocity u_0 .

Capacity of this settler is defined as the maximum of flow rate W at which all the targeted particles fall down to the tray while the fluid flows through.

From all the particles belonging to the targeted fraction, particles with terminal falling velocity u_0 entering the channel at height H need the longest time to settle out, namely

$$t_s = \frac{H}{u_0}$$

The speed of the gas in the channel is, by the law of continuity,

$$u = \frac{W}{A_{cross}} = \frac{W}{bH}$$

Thus, the residence time of the fluid in the channel is

$$\tau = \frac{L}{u} = \frac{LbH}{W}$$

All the targeted particles settle out if $t_s \leq \tau$, i.e. if

$$\frac{H}{u_0} \leq \frac{LbH}{W}$$

$$W \leq (Lb)u_0 = Au_0$$

Thus, the capacity of such a settling channel is proportional to its base area and nothing else.

Capacity of a settling centrifuge. As a result of the high field, the flow is normally laminar. In the *Stokes* region

$$u_0 = \frac{\Delta\rho d^2}{18\eta} g_c$$

and the field g_c depends on the turning rate $g_c = r\omega^2 = 4\pi^2 n^2 r$ where r is the actual distance from the turning axis of the centrifuge. Thus, the field depends on the actual radius which is smaller than the radius R of the drum, and changing during the operation because the particles gradually fill in the drum as is shown in **figure *****.

Specifying two radii r_1 and r_2 as start and end points, the time needed to reach from start to end is

$$T = \int_0^T dt = \int_{r_1}^{r_2} \frac{1}{u(r)} dr = \frac{18\eta}{\Delta\rho d^2 4\pi^2 n^2} \int_{r_1}^{r_2} \frac{1}{r} dr = \frac{18\eta}{\Delta\rho d^2 4\pi^2 n^2} \ln \frac{r_2}{r_1}$$

If V denotes the volume that can be carried on the centrifuge in one charge the capacity, i.e. the maximum flow rate W is

$$W = \frac{V}{t} = \frac{\Delta\rho d^2 4\pi^2 n^2}{18\eta \ln \frac{r_2}{r_1}} V = \left(\frac{\Delta\rho d^2 g}{18\eta} \right) \left(\frac{4\pi^2 n^2 V}{g \ln \frac{r_2}{r_1}} \right) \equiv u_0 \cdot A_e$$

where

$$A_e \equiv \frac{4\pi^2 n^2 V}{g \ln \frac{r_2}{r_1}}$$

is the settling area equivalent to a gravitational settler (a settling tray).

2.5 Sedimentor devices

Gases

For settling out dust from gases, a simple **gas chamber** are shown in **figure *****.

For settling out a large amount, **tray chambers** are applied as shown in **figure *****.

There are several versions of chambers based on collision and sudden redirection of the gas, shown in **figures *****.

Similar principles are applied in **drop settlers** (**figures *****). Special slanted and curved packings are also applied. Here the gas flows upward, and the liquid drops cannot follow the turns of the gas but slow down on the surface and then slip downward.

Cyclon is shown in **figure *****. The gas flows in tangentially. Whereas the gas turns and flows out in th chimney, of the smallest resistance, the heavier particles ($\rho_p \sim 1000\rho_f$) are by their inertia pressed to the wall; there they loose the kinetic energy, and slowly slip down.

Liquids

Rheo scrubbers are applied to separate lighter and heavier solid pieces. The solid mixture moves in a channel, and the Rheo units are placed under the channel in a series (**figure *****). Washing liquid (water) is introduced to the scrubber to lift up the lighter solid; the heavier solid falls down.

Dorr thickener is a conical device (**figure *****) for settling large amout of sludge. They are usually very wide, up to diameter 200 m. The wall is made of concrete for such a case, used for municipal sludge or minery. A scraper moves slowly (e.g. 0.02 turn/min) at the bottom. Fresh water is fed near the turning axis at the top, the sludge is removed at the bottom, and cleaned water is removed as overflow at the side.

Hydrocyclons are narrower and longer than cyclons. The liquid is introduced with high speed to reach large field because the density difference between the particles and the liquid is usually not high. For example, if the liquid is introduced with speed $u = 20$ m/s to a hydrocyclon on radius $r = 0.4$ m (a little wider than two feet), the the centrifugal field is

$$g_c = \frac{u^2}{r} = \frac{400\text{m}^2/\text{s}^2}{0.4\text{m}} = 1000 \frac{\text{m}}{\text{s}^2}$$

Considering the gravitation as $g \approx 9.81 \approx 10 \frac{\text{m}}{\text{s}^2}$, the centrifugal index is $Z \approx 100$.

Several hydrocyclons are applied in series for enhancing the efficiency of settling, but their number cannot be many because of the great loss of energy (pressure drop in the hydrocyclon).

Centrifuges

Several kinds of centrifuges for separating suspensions and emulsions are known. Some of them (Chamber fuges, tray fuges) are shown in **figures *****. Superfuges are longer and narrower devices.

Chapter 3

Packed columns and fluidization

Flow through tubes or columns packed randomly with solid pieces, or with structured packings, are applied in several unit operations. Such operations are those in which large phase contacting surface is to be provided in a given volume, such as adsorption, absorption, distillation, heterogeneous catalysis, as well as operations where the process material constitutes the packing, such as drying or drug extraction. Knowledge on the resistance against the flow through such packed tubes is of great importance.

Fluidized beds are also of great importance in drying, heterogeneous catalytic processes, and can even be used for absorption. A related field is pneumatic conveying.

3.1 Characterization of the packing

Consider a vertical tube (column) as shown in **figure *****, with internal diameter D , filled with packing up to height L over a horizontal grid. The following parameters are used in the discussion:

A	(total) cross section area (of the empty column), $A = \frac{D^2\pi}{4}$
V	(total) volume of the packing, $V = \frac{D^2\pi}{4}L$
V_{free}	free volume, (not occupied by the solid particles)
ε	voidage , $\varepsilon \equiv \frac{V_{free}}{V}$
A_{free}	free cross section area (not blocked by particles)
V_1	volume of a single particle
N	the number of particles
V_p	total solid volume, $V_p \equiv N \cdot V_1 = V - V_{free}$
ρ_1	density of a particle
ρ_b	density of the bed (total solid mass / total volume)
A_1	surface of a single particle
ω_1	specific surface of a single particle, $\omega_1 \equiv \frac{A_1}{V_1}$
ω	specific surface of the packing (total surface / total volume)

By average,

$$\frac{A_{free}}{A} = \frac{V_{free}}{V} = \varepsilon$$

In the same way,

$$\rho_b = \frac{N(\rho_1 V_1)}{V} = \rho_1 \frac{V_p}{V} = \rho_1 \cdot (1 - \varepsilon)$$

$$\omega = \frac{N \cdot A_1}{V} = (1 - \varepsilon) \omega_1$$

3.2 Flow and pressure drop

Speed of the flow with flow rate W through an **empty** column is denoted by and calculated as

$$u_0 = \frac{W}{A}$$

The actual speed in the narrow channel of the packing, by the law of continuity, is

$$u = \frac{W}{A_{free}} = \frac{W}{\varepsilon A}$$

Thus, the actual speed is

$$u = \frac{u_0}{\varepsilon}$$

and the *Reynolds* number is

$$Re = \frac{D_e u \rho}{\eta} = \frac{D_e u_0 \rho}{\varepsilon \eta}$$

For computing the equivalent (hydraulic) diameter, the internal circumference of the column (the tube), $D^2\pi$, may be neglected beside the wet circumference of the packing:

$$D_e = 4 \frac{A_{free}}{\text{circumference of packing}} = 4 \frac{V_{free}}{N \cdot A_1} = 4 \frac{\varepsilon V}{\omega V}$$

Thus, the equivalent diameter generally is

$$D_e = 4 \frac{\varepsilon}{\omega} = 4 \frac{\varepsilon}{(1 - \varepsilon) \omega_1}$$

and the *Reynolds* number is

$$Re = 4 \frac{u_0 \rho}{(1 - \varepsilon) \omega_1 \eta}$$

For **spherical particles** (balls) with diameter d , the specific surface is

$$\omega_1 = \frac{d^2 \pi}{\frac{d^3 \pi}{6}} = \frac{6}{d}$$

Substituting this value to ω_1 , one gets for spherical particles:

$$D_{e,sph} = \frac{2}{3} \frac{\varepsilon}{1 - \varepsilon} d$$

$$Re_{sph} = \frac{2}{3} \frac{d u_0 \rho}{(1 - \varepsilon) \eta}$$

For practical applications, however, simply

$$Re_p = \frac{d u_0 \rho}{\eta}$$

is also used for packings, and one has to check which definition is applied in an empirical or approximating formula.

Laminar flow

Substituting into the *Hagen–Poiseuille* law

$$\Delta p = \frac{32 L \eta u}{D_e^2}$$

one gets

$$\Delta p_{sph} = 72 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{L \eta u_0}{d^2}$$

However, **Blake and Kozeny** empirically found

$$\Delta p_{sph} = 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{L \eta u_0}{d^2}$$

The empirically larger coefficient can be explained by considering dense spherical packing and, therefore, strong curvature of the channels in the packing. According to experiments, the **Blake–Kozeny equation** is valid if $\varepsilon < 0.5$ and $Re_p < 10$.

Turbulent flow

The general formula of pressure drop is

$$\Delta p = f \frac{L}{D_e} \frac{u^2 \rho}{2}$$

One gets by substitution

$$\Delta p = f \frac{3}{2} \frac{(1 - \varepsilon)}{\varepsilon^2} \frac{u^2 \rho}{2} \frac{L}{d} = 0.75 f \frac{(1 - \varepsilon)}{\varepsilon^3} u_0^2 \rho \frac{L}{d}$$

Experiments, however, show the effect of the curvature of the channels in the packing, and the **Burke–Plummer equation** can be used instead:

$$\Delta p = 1.75 f \frac{(1 - \varepsilon)}{\varepsilon^3} u_0^2 \rho \frac{L}{d}$$

This is valid if $Re_p > 1000$.

General case

A unified approximating formula is given by **Ergun**:

$$\Delta p = u_0^2 \rho \frac{(1 - \varepsilon)}{\varepsilon^3} \frac{L}{d} \left[\frac{150 (1 - \varepsilon)}{R_p} + 1.75 \right]$$

or, in a dimensionless form

$$\frac{\Delta p}{u_0^2 \rho} = Eu = \frac{(1 - \varepsilon)}{\varepsilon^3} \frac{L}{d} \left[\frac{150 (1 - \varepsilon)}{R_p} + 1.75 \right]$$

3.3 Fluidized bed

Consider a vertical column partially filled with packing resting on a grid, and fluid is driven through it from below. By increasing the flow rate, the resistance, and thus the pressure drop, increases monotonically. However, if the packing is not restricted from above by another grid then the pressure drop stops increasing at

a certain flow rate, and remains constant in a wide range of flow rate above this value. Instead of an increase in the pressure drop, the packing floats over the grid, and its height increases with the flow rate. It is more instructive to consider the speed u_0 instead of the flow rate W . The fluidization is maintained between a lower speed u_0^* and a higher speed u_0^{**} . Meanwhile the height L of the packing is not constant. For general discussion, an L_0 , so-called **height of dense packing**, can be introduced as $L_0 = \varepsilon L$; this remains constant as a property of the system. L_0 can be imagined as the height the packing would take if it were melted. A general **fluidization plot** is shown in figure ***.

When the fluidization starts can be calculated by equilibrium of forces. When the fluid does not move ($u_0 = 0$), the packing presses the grid with its Archimedian weight: $G_A = (L_0 A) (\rho_1 - \rho) g$, and the pressure on the grid is $p_{grid} = \frac{G_A}{A} = L_0 \Delta \rho g$. This pressure is constant; it depends on the material properties only. The pressure drop one can measure between the two ends of the packing can be interpreted as the drag force related on the total cross section: $\Delta p = \frac{F_{drag}}{A}$. This drag force, and thus the pressure drop Δp can be calculated e.g. by the **Ergun** equation.

As u_0 increases, the pressure drop and the drag force also increases and thus the pressure on the grid from above decreases. At some speed u_0^* the two forces just balance each other, $\Delta p = p_{grid}$, and the packing floats.

According to *Newton's* law, the packing's particles ought to accelerate upward and fly out from the column if $u_0 > u_0^*$. However, it is not u_0 what actually is involved in the drag force but some speed u in the free cross section inside the packing; this is $u = \frac{A}{u_0}$. As soon as the packing starts floating, and lifts up, its particles arrange looser than earlier. The packing bed becomes higher (L increases), and thus A_{free} and ε also increase. The packing opens up just so much as to maintain the balance of forces. With a smaller ε it would fly out from the column because the drag force were higher than the Archimedian weight; with a higher ε it would fall down because the Archimedian weight were higher than the drag force. Over all the range of fluidization, the drag force is just equal to the Archimedian weight; this is why the pressure drop is constant.

This sedimentation balance can be maintained as far as there is some way to loose up the packing, i.e. as far as ε does not approach 1. When there is no more but a single particle in a cross section in average, the cross section cannot be increased without carrying out the particles by the fluid. This happens above u_0^{**} . Just at u_0^{**} , the single particles float in the fluid; thus u_0^{**} is the terminal speed of settling in infinite space, and can be calculated accordingly.

The **Ergun** equation is just an approximation; experimental plots can be constructed for particular packings. For avoiding iterative graphical procedure, $f Re^2$ is plotted against Re because $f Re^2$ does not depend directly on u_0 . Along with defining Re_p as above, a modified f_p is also usually defined as

$$\frac{2}{3} f \frac{1}{\varepsilon^3} \equiv 4f_p$$

so that

$$\Delta p = f \frac{3}{2} \frac{(1 - \varepsilon)}{\varepsilon^3} \frac{u_0^2 \rho}{2} \frac{L}{d}$$

becomes

$$\Delta p = 4f_p (1 - \varepsilon) \frac{u_0^2 \rho}{2} \frac{L}{d} = 4f_p \frac{u_0^2 \rho}{2} \frac{L_0}{d}$$

In balance of forces

$$\boxed{4f_p \frac{L_0}{d} \frac{u_0^2 \rho}{2} = L_0 \Delta \rho g}$$

so that

$$f_p = \frac{\Delta \rho g d}{2\rho u_0^2}$$

and

$$f_p Re_p^2 = \frac{g \Delta \rho d \rho}{2\eta^2}$$

A chart for $f_p Re_p$ against Re_p with varying ε voidages for spherical particles in air is shown in **figure *****. Such a chart is applicable for modelling fluidization only. For a given speed u_0 and voidage ε , first $f_p Re_p$ can be read, and then it can be divided by Re_p to obtain f_p , and then computing Δp . The voidage of the packing in still (before fluidizing) can be measured, and thus u_0^* determined. But the chart is valid all over the fluidization range.

Inhomogeneous and homogeneous fluidization. The fluidization can be inhomogeneous, i.e. there are channels in the bed, and the bed is pulsing, if $Fr > 1$. This is usually the case if the fluid is gas. At $Fr < 1$, homogeneous fluidized bed is experienced; this is usually the case if the fluid is liquid.

Hysteresis. When particles with irregular shape are first fluidized, the pressure drop first increases a little bit higher and then drops back to the constant pressure drop of the fluidization range (**figure *****). When afterwards decreasing the speed, this bump is not experienced; neither can it be experienced in the second fluidization experiment. This phenomenon can be explained by rearrangement of the particles along with the flow.

Carry out. Above u_0^{**} , the particles are carried over by the fluid. Considering a very long pipe, such a situation is *pneumatic conveying*, and the pressure drop increases with u_0 . Considering a short column, the particles are carried out from the system, and the pressure drop decreases in time until the column becomes free of the packing, and the pressure drop of the empty column is experienced. This empty column pressure drop then again increases with u_0 .

Chapter 4

Filtration

4.1 Batch arrangement

Filtration is separation of solid particles from a fluid by letting the fluid through narrow capillary channels which prevent the solid particles to go through. The basic arrangement of filtration is shown in **figure *****. The filter itself is a channel with a grid that merely serves as a holder for a *filter cloth* spanned on it. The suspension is fed to the channel from the cloth side. The clear fluid free of the solid particles flows through to the other side, and is called the **mother liquor** or **filtrate**.

The solid particles are kept in the input side. The potential of the cloth to keep back the solid particles is usually small. However, once a layer of retarded solids is formed on the cloth, this sludge layer performs most of the effect of filtration. The capillary channels in this so-called **filter cake** prevents the particles from going through.

On the other hand, the filter cake gives rise to a large resistance against the flow of the mother liquor as well. The force to drive the flow through the cake can be provided by gravitation, pressure on the feed side, vacuum on the filtrate side, and their combination. Instead of gravity, higher field can be achieved by centrifuge.

Filtering rate W is defined as the flow rate of the filtrate, and is an important industrial characteristic of the process. **Filtering velocity** is $u = \frac{W}{A}$ where A is the surface of the filter (grid, or the internal diameter of the channel, in our example). This rate or velocity depends on the resistances forming in the device.

There are three main constituents of resistance: (1) Δp_d of the filter device itself together with all the fittings, (2) Δp_f of the filter cloth, and (3) Δp_c of the filter cake. The pressure drop caused by the device itself can usually be neglected, and is lumped with the resistance of the filter cloth to give a resistance of media: $\Delta p_m \equiv \Delta p_d + \Delta p_f$. Even Δp_m may be neglected comparing to Δp_c , but this member is sometimes taken into account.

Auxiliaries. Beside filter cloth and washing water, other auxiliary materials are sometimes used for enhancing the process if the suspension too easily goes through the cloth. These materials include mechanical auxiliaries with high specific surface like carbon powder, perlite, sawdust, and coagulating chemicals.

4.2 Resistance and basic equation

Resistance in the cake is caused by the narrow pores; thus, the *Hagen–Poiseuille* equation can be applied because the flow in the pores is always laminar. One can start with the *Blake–Kozeny* equation because the sludge can be considered as a dense packing. These equations can be expressed in this case as

$$\Delta p_c = 150 \frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{L \eta \omega_1^2 u_0}{36}$$

where the $d\omega_1 = 6$ is substituted. From here

$$u_0 = B \frac{\Delta p_c}{\eta L}$$

where

$$B \equiv \frac{36 \varepsilon^3}{150 (1 - \varepsilon)^2 \omega_1^2}$$

is a lumped parameter called *permeability factor* with dimension m^2 , and is constant during the process. This equation for u_0 is due to *Darcy*, and is called the **Darcy equation**.

From the *Darcy* equation another form can be derived for the pressure drop by rearrangement, because $W = u_0 A$:

$$\Delta p_c = \frac{\eta L}{B} \frac{W}{A} = \frac{\eta V}{B A^2} W$$

where V is the volume of the cake. It can also be expressed as

$$\Delta p_c = \alpha \frac{\eta c V}{A^2} W$$

where c is the solid mass cumulated in the cake related to the filtrate volume [kg solid / m^3 filtrate], and α is a so-called specific cake resistance

$$\alpha = \frac{150 (1 - \varepsilon) \omega_1^2}{36 \varepsilon^3 \rho_1}$$

Value of α depends on the *porosity* ε which can be considered constant in many cases. It occurs, however, in some cases that the porosity depends on the pressure pressing the cake, and even on the height of the cake. In this case we speak about

compressible cake. In this compressible case α may vary along with the height of the cake, but an average α may be used for approximating calculations.

Pressure drop on the media can be modelled in analogy to the cake, by substituting the $\alpha cV/A$ factor with a so-called *resistance of media* R_m [1/m]:

$$\Delta p_m = \frac{\eta}{A} R_m W$$

resulting in the **Carman equation** of filtration:

$$\Delta p = \frac{\eta}{A} \left(\alpha c \frac{V}{A} + R_m \right) W \equiv \frac{\eta}{A} \left(\alpha c \frac{V}{A} + R_m \right) \frac{dV}{dt}$$

4.3 Filtration time

The *Carman* equation can be readily integrated if the pressure difference is constant:

$$\begin{aligned} dt &= \frac{\eta}{\Delta p} \left(\frac{\alpha c}{A^2} V + \frac{R_m}{A} \right) dV \\ t &= \frac{\eta}{\Delta p} \left[\frac{\alpha c}{2} \left(\frac{V}{A} \right)^2 + R_m \left(\frac{V}{A} \right) \right] \end{aligned}$$

This equation gives t as a second order polynomial of V . In practice, the $V(t)$ function is looked for and measured; this is an increasing function with decreasing slope:

$$V = \frac{A}{\alpha c} \sqrt{\frac{2\alpha c \Delta p}{\eta} t + R_m^2} - R_m$$

The function has two parameters characterizing the material and the media: α and R_m . Once these parameters are determined by small scale (laboratory) experiments, the same values can be applied to design an industrial scale process.

For determining the parameters by experiment, $\frac{dt}{dV}$ values, or their approximation $\frac{\Delta t}{\Delta V}$ are plotted against V (**figure *****). This plot should give an approximating straight line because

$$\frac{\Delta t}{\Delta V} \approx \frac{dt}{dV} = \frac{\eta \alpha c}{A^2 \Delta p} V + \frac{\eta R_m}{A \Delta p} \equiv a V + b$$

where a is the slope of the straight line, and b is its intersection with the axis.

Optimal filtering time. There is also a changeover time τ in batch processes; this is the time used for washing and removing the filter cake, and then washing the device and, finally, fitting new cloth, etc. Considering several consecutive batch turns, the productivity of the process is measured by

$$J_V = \frac{V}{\tau + t}$$

The changeover time being constant, there is an optimal V (and an optimal t) where the process ought to stop for maximizing the productivity. This is illustrated in **figure *****. Instead of $V(t)$, another $V'(\tau + t)$ function is plotted so that $V'(\tau + t) = V(t)$. The productivity is at maximum if the straight line drawn from the starting point (the origo) and tangent to $V'(\tau + t)$ has maximum slope. This can also be determined analytically, as

$$V_{opt} = A \sqrt{\frac{2\Delta p \tau}{\eta \alpha c}}$$

$$t_{opt} = \tau + R_m \sqrt{\frac{2\eta \tau}{\alpha c \Delta p}}$$

Usually $t \approx \tau$ or $t \approx 1.2 \sim 1.25\tau$

Filtering with constant rate. Should a constant filtering rate W be produced, the pressure must be continuously increased during the batch process, and must follow a calculated function. By rearranging the above equations, Δp as a function of V or t is linear:

$$\Delta p = \frac{\eta W}{A} \left(\frac{\alpha c}{A} V + R_m \right) = \frac{\eta W}{A} \left(\frac{\alpha c}{A} W t + R_m \right)$$

4.4 Filter devices

Liquid sieve. Liquid sieves are rough devices for filtering large scale solid pieces.

Filter tub. Devices of tub shape, for batch filtration, working with gravity, pressure, or vacuum. Vacuum driven filter tubs are sometimes called **Nutsch filters**, shown in **figure *****.

Filter bed. These are tanks or tubs filled with pebbles (**rubble filter, figure *****) or sand (**sand filter, figure *****). They are usually open to air and serve for slow filtering (~ 0.1 m/h), driven by water level difference, slow enough to save the bed from deformation or channel formation, and to provide time for accomplishing preferred bioprocesses. They are characterized by large space and cost. Closed tanks filled with sand bed (1.2 to 3 m height) can be used for fast filtering (7 to 15 m/h).

Filter press. This is a batch device made of a series of uniform element pairs (frames) that are assembled for filtering and disassembled for cleaning, as shown in **figures *****. The elements are fitted with filter cloth and then pressed together for filtering. Alternating elements are put in the series so that small chambers for accumulating the cake and others for channeling the liquid are connected with

small holes that are assembled to form tubes. The suspension is led to the devices by pressure, so that the pressure is higher inside the device than its environment where the filtrate is collected.

Pressure filter. Contrary to the filter press, here the series of devices are put in a pressurized vessel containing the suspension, and the filtrate forms inside the smaller devices covered by filter cloth. The filtrate is removed through collector tubes. One form is the **filter bags** which usually hang on a common rail inside a horizontal vessel (**figure *****), and their form is protected by wire grid against being pressed flat. Another form is the **filter fingers** or **filter candles** which form a kind of tube bundles inside a vertical vessel (**figure *****). The real difficulty in the pressure filters is removing the cake. There are several applicable techniques including washing, drying with gas, centrifugal effect, etc.

Filter drum. Filter drum is a rotating device for continuous filtering (**figure *****). A drum is rotated slowly around a horizontal axis, and a part of the drum is submerged in a riffle filled with the suspension. Filter cloth is fitted to the superficies of the drum, and vacuum is maintained inside so that the filtrate forms inside the drum and the cake forms on the cloth from outside. The suspension is continuously re-filled to maintain a constant level in the riffle. The cake is continuously removed by a blade from the upper side of the drum. This cake removal is usually enhanced by pressing it from inside. For this aim, the drum is divided to compartment sections from inside, and which section is pressed or sucked is switched according to the rotation. Sophisticated devices include sections for sucking, washing, drying, loosening, pressing.

Productivity of a filter drum is determined by the average width of the cake forming on the drum, and the time one point of the filter cloth spends in the suspension in one turn of the drum.

A version of filter drum filters in reverse direction: from inside to outside. They are used if the suspension liquid also contains solid particles which settle too fast and cannot be kept in suspension.

Filter belt. This is another continuous filtering device (**figure *****). A close belt fitted with filtering medium moves continuously; the suspension is fed over it at a point above, the filtrate is collected below; the cake formed on the belt is washed at a later point, and then removed at the end of the horizontal section.

Tray filter. This is a slowly rotating horizontal disk containing a series of flat filter trays or chambers (**figure *****). This is not a real continuous process but a series of batch processes in circle. However, it can be applied to very dense suspensions which cannot be filtered with drum.

Centrifuges. They work in the same principles as the centrifuges in households. Some filtering and settling fuges are shown in (**figures *****).

Chapter 5

Mixing

5.1 Mixers

Mixers, stirrers, agitators, impellers, scrapers, kneaders are all rotating devices submerged in some liquid to be mixed, stirred, etc. They are all fitted to a shaft rotated by a motor. There are a numerous forms of fittings rotated in the liquid; some basic classes are listed below.

The vessel in which the impellers rotate may also be fitted with baffles as is shown in **figure ***** in order to make a resistance against easily circulating the liquid along the wall and without strong mixing, and also to prevent the liquid from forming a funnel in the center and sucking gas from over the level. Sucking gas into the liquid enhances foaming.

Paddle stirrer, arm stirrer, grid and horseshoe stirrers. As shown in **figures *****, these basic devices are simple blades or plates parallel with the shaft. They can be long or short compared to the internal diameter D of the vessel; this length of the blade or arm is called the diameter of the mixer, and denoted in the present text by d . Width w of the blade is its height along the shaft. If d/D is small and w comparable to d then we speak about plate stirrer; if d/D is large and w/d is small then we speak about arm stirrer. Distance h of the lower edge of the blade from the bottom is also an important property of the system. Several arms can be fitted on the shaft above each other. The turning number is usually 20 to 40 /min.

A version of this flat type stirrers is stock or grid stirrers which consist of vertically arranged blades near the wall.

Paddle stirrers can be made with two or three blades at the same height arranged in star form on the shaft.

Horseshoe stirrers are also called anchor (or anker) stirrers, as well, and they can also be made in grid version.

Helical-blade stirrer. Such devices (**figures *****) are applied for strong mixing, and careful homogenization.

Propeller mixer, compressor impeller. These devices (**figures *****) consist of curved surfaces similar to the ship screw, and push the liquid along the direction of the shaft. They are used for large amount of liquid, with great turning number.

Turbine mixer, centrifugal impeller. These are very fast turning devices shown in **figures *****. They suck in the liquid from above and/or from below, and push them out at the superficies; and thus make a strong circulation of the liquid.

Knealers. Some types are shown in **figures *****.

5.2 Power consumption

Power consumption P [J/s] may depend on the density ρ [kg/m³] and viscosity η [kg/(m s)] of the liquid, the gravity present g [m/s²], the turning number n [1/s], and geometrical data such as D , H , d , w , h , ar even more such length is they characterize the vessel and the impeller. A polynomial form can be written as

$$P = B\eta^a g^b D^c \rho^e n^f d^i H^j w^k h^s$$

where B is an unknown dimensionless constant factor. The exponents of the dimensions should satisfy the equations for kg, s, and m, respectively:

$$\begin{aligned} 1 &= a + e \\ -3 &= -a - 2b - f \\ 2 &= -a + b + c - 3e + i + j + k + r \end{aligned}$$

Three exponents are expressed in function of the others:

$$\begin{aligned} e &= 1 - a \\ f &= 3 - a - 2b \\ i &= 5 - 2a + b + c - j - k - r \end{aligned}$$

so that

$$P = B\eta^a g^b D^c \rho^{1-a} n^{3-a-2b} d^{5-2a+b+c-j-k-r} H^j w^k h^s$$

that can be rearranged according to the exponents as

$$P = \rho n^3 d^5 B \left(\frac{\eta}{d^2 n \rho} \right)^a \left(\frac{g}{dn^2} \right)^b \left(\frac{D}{d} \right)^c \left(\frac{H}{d} \right)^j \left(\frac{w}{d} \right)^k \left(\frac{h}{d} \right)^q$$

The dimensionless *Euler* number of mixing and the *Reynolds* number of mixing are defined as

$$Eu = \frac{P}{\rho n^3 d^5}$$

$$Re = \frac{d^2 n \rho}{\eta}$$

so that a dimensionless equation can be written:

$$Eu = B \left(\frac{D}{d} \right)^c \left(\frac{H}{d} \right)^j \left(\frac{w}{d} \right)^k \left(\frac{h}{d} \right)^q Re^{-a} Fr^{-b}$$

The *Reynolds* number of mixing is obtained from the original definition $Re = (du\rho)/\eta$ by substituting the circumferential speed of the impeller $u = dn$. The power consumption P can be expressed as the force the blade exerts on the liquid multiplied by the speed it moves. The force is proportional to the pressure it exerts on the liquid and the area the blade touches: $P \sim \Delta p d^2 u$. Thus

$$Eu = \frac{P}{\rho n^3 d^5} = \frac{\Delta p d^2 u}{\rho u^3 d^2} = \frac{\Delta p}{u^2 \rho}$$

This is why we speak about *Euler* number of mixing.

For a given geometrical arrangement, i.e. fixed ratios of D/d , H/d , etc., the geometric ratios can be lumped with the constant factor to give

$$Eu = A Re^{-a} Fr^{-b}$$

This equation can be applied to scale-up at design because Eu , Re , and Fr depend on the material properties and the rotation number only.

The exponent b is usually negligible. Its effect is essential only if the mixing is so fast that air (or other gas) is sucked to the liquid near the shaft.

A typical $Eu-Re$ plot is shown in **figure *****. At slow mixing, i.e. laminar flow, $a = 1$, and $b = 0$. This is experienced if $Re < 10$. If there are side baffles in the vessel then $a = 0$ in the turbulent region. Otherwise, $a \approx 0.1 \sim 0.2$.

Note that the above data makes possible to estimate the effect of geometric scaling up or speeding up the rotation if geometric similarity is maintained.

In laminar region. Here $a = 1$, and thus

$$P = A \rho n^3 d^5 \frac{\eta}{d^2 n \rho} = A \eta n^2 d^3$$

That is, the power consumption is quadratic in the rotation number, and cubic in the diameter.

In turbulent region. Here $a \approx 0$, and thus

$$P \approx A \rho n^3 d^5$$

That is, the power consumption is cubic in the rotation number, and of fifth degree (!) in the diameter. This is why mixing with long arms is so difficult.

Chapter 6

Heat transport

Heat itself is the change or transport of internal energy or enthalpy, depending on the case.

There are three kinds of heat transport: conduction, convection, and radiation. Heat conduction is transport of internal energy, driven by temperature difference or slope. Heat convection is carrying internal energy in space by moving material. Radiation is transport of internal energy mediated by electromagnetic waves through vacuum or dilute gases.

Heat transport is a sloppy expression for transport of internal energy.

6.1 Heat conduction

A one-dimension version of **Fourier's** law of heat conduction is

$$Q = -\lambda A \frac{dT}{dx}$$

where Q is **heat transport** [W] in direction x , A is area [m²] of the surface perpendicular to x , through which the energy is transported, $T(x)$ is temperature [K] or [C] homogeneous in planes perpendicular to x but dependent on space coordinate x [m], and the factor λ [W/(m K)] is a material property called **heat conductivity**. The negative sign indicates that heat is transported toward lower temperature. A version of the equation is

$$q = -\lambda \frac{dT}{dx}$$

where

$$q \equiv \frac{Q}{A}$$

is **heat flux** [W/m²].

Heat conduction through homogeneous planar wall. *Fourier's law* is a differential equation that can be analytically integrated only if λ and A are special functions of x . Such a special case is when both λ and A are constant. Consider a planar wall of width w and homogeneous with constant heat conductivity λ . Suppose constant temperature T_1 all over one side of the wall, and constant temperature $T_2 < T_1$ all over the other side of the wall (**figure *****). Because of symmetry, and constant A along direction x perpendicular to the wall plane, one may consider the heat flux form of the equation. How large A is unimportant; one may consider an arbitrary small A neglectable comparing to w , i.e. an arbitrary narrow straight rod. It follows that the heat flux q is everywhere in the wall is directed along x , and zero in perpendicular directions.

In steady state, i.e. when T and q are constant in time, q must be constant along x ; otherwise energy would accumulate or disappear from some space sector, and this would involve temperature increase or decrease, contrary to the steady state. Thus q and λ can be set off the integrals so that

$$\begin{aligned} q \int_0^w dx &= \lambda \int_{T_1}^{T_2} dT \\ q w &= \lambda (T_1 - T_2) \\ q &= \frac{\lambda}{w} (T_1 - T_2) \end{aligned}$$

The integration can be performed between any two planes, e.g.

$$\begin{aligned} q \int_0^x d\xi &= \lambda \int_{T_1}^T d\vartheta \\ q x &= \lambda (T_1 - T) \end{aligned}$$

which clearly shows that T is linear along x :

$$T = T_2 - \frac{q}{\lambda} x$$

Another lesson drawn from this result is that the reciprocal of λ is a kind of thermal resistance.

Heat conduction through composite planar wall. Consider a composite wall consisting of three homogeneous planar wall sections of the same cross section area A and touching pairwise at the planar sides (**figure *****). Let the widths and conductivities be w_1, w_2, w_3 , and $\lambda_1, \lambda_2, \lambda_3$, respectively. Let the temperatures at the sides be, in the same order, $T_1, T_{1,2}, T_{2,3}$, and T_2 . Each section can be computed as above and, in the same way, the heat flux q must be the same everywhere. The

temperature differences can be expressed as

$$\begin{aligned}\frac{w_1}{\lambda_1} q &= T_1 - T_{1,2} \\ \frac{w_2}{\lambda_2} q &= T_{1,2} - T_{2,3} \\ \frac{w_3}{\lambda_3} q &= T_{2,3} - T_2\end{aligned}$$

These equation can be added to give

$$\left(\frac{w_1}{\lambda_1} + \frac{w_2}{\lambda_2} + \frac{w_3}{\lambda_3} \right) q = T_1 - T_2$$

so that

$$\begin{aligned}q &= \frac{1}{\frac{w_1}{\lambda_1} + \frac{w_2}{\lambda_2} + \frac{w_3}{\lambda_3}} (T_1 - T_2) \\ Q &= \frac{A}{\frac{w_1}{\lambda_1} + \frac{w_2}{\lambda_2} + \frac{w_3}{\lambda_3}} (T_1 - T_2)\end{aligned}$$

with a lesson that resistances in series are added together.

Heat conduction through circular walls. Here A is not constant along the radius r but $A = 2\pi rL$ where L is the length of the tube (**figure *****). It follows that q cannot be constant either. Thus, the original form is applied:

$$Q = -\lambda 2\pi L r \frac{dT}{dr}$$

In the same way as earlier, Q must be constant along r , and the equation is integrated as

$$\begin{aligned}Q \frac{1}{r} dr &= -\lambda 2\pi L dT \\ Q \ln \frac{r_2}{r_1} &= \lambda 2\pi L (T_1 - T_2) \\ Q &= \frac{\lambda 2\pi L (T_1 - T_2)}{\ln \frac{r_2}{r_1}}\end{aligned}$$

If $r_2 - r_1$ is small compared to r_2 then the conduction can be approximated with planar wall.

Integration for composite circular wall leads to

$$Q = \frac{\lambda 2\pi L (T_1 - T_2)}{\sum_i \left(\frac{1}{\lambda_i} \ln \frac{r_{i+1}}{r_i} \right)}$$

6.2 Heat convection and film theory

Film coefficient

Heat convection cannot exist without simultaneous heat conduction because the latter always occurs if there is temperature slope or difference in space.

Prandtl developed a model (or theory) to describe heat transport between solid wall surface and liquid or gas. Imagine, for example, the case when the planar wall surface is warmer than the fluid so that heat is transported from the wall to the fluid. Depending on how the fluid beside the wall is mixed, the temperature decreases faster or slower toward the bulk of the fluid, in a continuous temperature curve along x distance from the wall (**figure *****).

There is always a laminar layer at the wall through which the heat is conducted only. The temperature dependence along x in the laminar layer is linear, according to the preceding section. The slope of $T(x)$ in the layer is identical to the slope at $x = 0$ or $x \rightarrow 0$. *Prandtl's* model extends the laminar layer to wider film with the effective width δ_{eff} so much that at $x = \delta_{eff}$ the temperature just reaches the temperature in the bulk of the fluid (**figure *****).

This model approximates the real process in which convection and conduction occur parallel with a model of conduction and convection in series. From the wall till δ_{eff} pure conduction is considered with constant λ heat conductivity, with a linear temperature profile, and from δ_{eff} to any distance an infinitely fast heat convection is assumed that could be accomplished by perfect mixing only. This perfect mixing provides us with the constant temperature seen in the figure.

Thus, according to the model of *Prandtl*, all the resistance against the heat transport is concentrated in the effective laminar film, so that

$$q = \lambda \frac{(T_s - T_b)}{\delta_{eff}}$$

where T_s is the temperature at the surface of the wall, and T_b is the bulk temperature.

The effective width δ_{eff} is not known in practice. Conductivity λ of the fluid would be valid in the case of liquid in still, even free of the so-called natural convection induced by temperature difference, and used in a differential model. However, these two data can be combined in a single **film coefficient of heat transport** α [W/m²K] so that what the transport is proportional to is not the slope but the temperature difference:

$$q = \alpha (T_s - T_b)$$

$$\boxed{Q = \alpha A (T_s - T_b)}$$

α is not a material property but it depends also on the flow (or mixing) of the fluid. Its value is low if the fluid does not move, and high if there is fast motion perpendicular to the wall. On the other hand, α depends on the material properties as well through the dependence of λ .

Prandtl–Nusselt analogy

Instead of analytically or numerically solving the rather complicated system of differential equations describing the three dimensional transport of impulse and energy, dimensional analysis can be used to find an applicable numerical model.

Forced convection. When natural convection does not occur or can be neglected, the following variables should be taken into account for a system where a fluid flows in tube or along a wall:

α	film coefficient
λ	heat conduction
ρ	density
η	dynamic viscosity
c_p	specific heat
u	speed of the flow
D	a characteristic length, e.g. diameter of a tube
L	length of the device (e.g. of the tube)

The empirical dependence of α on the other variables is looked for in a polynomial form:

$$\alpha = A \rho^a \eta^b \lambda^c c_p^d u^e D^f L^h$$

Considering the dimensions, four equations follow for the exponents of mass, time, temperature, and length, respectively:

$$\begin{aligned} 1 &= a + b + c \\ 3 &= b + 3c + 2d + e \\ 1 &= c + d \\ 0 &= -3a - b + c + 2d + e + f + h \end{aligned}$$

Exponents a , b , c , and f can be expressed by e , d , and h as

$$\begin{aligned} a &= e \\ b &= d - e \\ c &= 1 - d \\ f &= e - h - 1 \end{aligned}$$

so that the polynomial is expressed as

$$\alpha = A \rho^e \eta^{d-e} \lambda^{1-d} c_p^d u^e D^{e-h-1} L^h$$

that can be rearranged according to the exponents as

$$\frac{\alpha D}{\lambda} = A \left(\frac{D u \rho}{\eta} \right)^e \left(\frac{c_p \eta}{\lambda} \right)^d \left(\frac{L}{D} \right)^h$$

that is

$$Nu = A Re^m Pr^n \left(\frac{L}{D}\right)^k$$

where

$$Nu = \frac{\alpha D}{\lambda}$$

is the **Nusselt number**, and

$$Pr = \frac{c_p \eta}{\lambda}$$

is the **Prandtl number**. This latter one can also be expressed as

$$Pr = \frac{c_p \eta}{\lambda} = \frac{\frac{\eta}{\rho}}{\frac{\lambda}{c_p \rho}} = \frac{\nu}{a}$$

where ν is the kinematic viscosity, and a is *heat diffusivity*.

Remembering the definition of α , it is clear that the *Nusselt number* Nu is the ratio of the characteristic length (D) to the effective width of the laminar film (δ_{eff}). The *Prandtl number* Pr is the ratio of viscosity to heat diffusivity, i.e. the ratio of moment transport to heat transport.

The values of the exponents in the analogy equations depend on the actual arrangement such as heat transport inside or outside a tube, along a wall, a tube bundle, mixed vessel, boiling, condensation, cooling/heating of gas or liquid, etc. Informative values are the following:

Forced flow, turbulent, in straight tube ($Re > 10^4$):

$$Nu \approx 0.023 Re^{0.8} Pr^{\frac{1}{3}}$$

Forced flow, laminar, in straight tube ($Re < 2300$):

$$Nu \approx 1.86 \left(Re Pr \frac{D}{L} \right)^{\frac{1}{3}}$$

However, these equations cannot be used directly because the material properties η , ρ , and c_p depend on T , and thus change their value in space with an essential scale. Viscosity may change decades with temperature. Thus, they are usually corrected with a factor taking into account the change of η between the bulk and the wall surface:

$$Nu \approx 0.023 Re^{0.8} Pr^{\frac{1}{3}} \left(\frac{\eta_b}{\eta_s} \right)^{0.14}$$

$$Nu \approx 1.86 \left(Re Pr \frac{D}{L} \right)^{\frac{1}{3}} \left(\frac{\eta_b}{\eta_s} \right)^{0.14}$$

Inside a helical tube, the heat transport is better because the centrifugal force makes the laminar layer smaller. One can approximate

$$\alpha \approx \alpha_{\text{straight}} \left(1 + 1.77 \frac{D}{R} \right)$$

where R is the radius of the coil.

Several empirical plots are available in the literature for either Nu or $Y \equiv \frac{Nu}{Pr^b}$ or $j_H \equiv \frac{Y}{Re}$ as function of Re .

Natural convection. When there is temperature difference between higher and lower fluid points, then the densities are also different, and this density difference may induce a vertical motion of the fluid. The warmer fluid rises and the cooler ascends. This motion is called *natural convection*.

In this case three additional factors should be taken into account:

- ΔT temperature difference
- β_T cubic coefficient of thermal expansion [1/K]
- g gravitational acceleration

Then the dimensionless equation is

$$Nu = A Re^m Gr^b Pr^n \left(\frac{L}{D} \right)^k$$

where Gr is the **Grashof number**:

$$Gr = \frac{\beta_T g \Delta T \rho^2 D^3}{\eta^2}$$

which expresses the ratio of buoyancy per volume to internal friction. When the motion is not forced but purely natural, Re can be neglected:

$$Nu = A Gr^b Pr^n \left(\frac{L}{D} \right)^k$$

When the gas or liquid flows along a vertical wall, the temperature difference is interpreted as $\Delta T = T_s - T_b$. The characteristic length D is interpreted according to the situation. In horizontal tubes with internal diameter d , the characteristic length is the average distance between the upper and lower inner wall.

Natural circulation usually forms between two vertical walls, like glass windows, is there is enough distance between them; this circulation enhances the heat transport. If the distance of the two windows is low (the double window is narrow) then no circulation can form, and the heat insulation is better.

Considering two horizontal walls, the situation depends on whether the upper or the lower wall is warmer. If the upper wall is warm, and the lower wall is cold, then there is no natural convection. In the other case, however, natural convection form, giving rise to circulation.

6.3 Heat convection at phase change

The heat transport related to condensation and boiling is usually much better than that of liquids or gases.

Condensation

The condensation as a process does not depend essentially on the temperature difference between the vapor (T) and the wall surface (T_s). It occurs even at very small, almost zero temperature difference if the vapor is at dew point temperature.

The film coefficient can be as high as 10 to 15000 [W/(m²K)].

When steam or other vapor flowing horizontally is condensed on vertical wall, the condensate accumulates on the wall as a layer with increasing width downwards because the layer slips down on the wall by gravity. In steady state, the weight of the layer is balanced by the friction force. Thus, the accumulation of mass along the vertical direction x can be expressed as a function of the width δ of the layer:

$$\frac{dm}{dx} = f(\delta(x))$$

In the same time, the heat produced by condensation in unit time (is equal to that transported through the condensate layer

$$\frac{dm}{dx} \Delta H = \frac{\lambda}{\delta(x)} b (T - T_s) = \alpha b (T - T_s)$$

where ΔH is the condensation heat, and b is the horizontal length of the wall. From here, the following equation can be derived for average film coefficient of condensation on vertical plane wall or vertical rod:

$$\alpha \approx 0.943 \left(\frac{\lambda^3 \rho^2 \Delta H g}{\eta L \Delta T} \right)^{\frac{1}{4}}$$

where L is the vertical length of the falling condensate film. It follows that the heat transport of such condensation can be enhanced by shortening L . This can be achieved by fitting the vertical tubes with circular rings at periods of 0.3 - 0.4 m.

For condensation on horizontal rods (outside of horizontal tubes):

$$\alpha \approx 0.728 \left(\frac{\lambda^3 \rho^2 \Delta H g}{\eta D \Delta T} \right)^{\frac{1}{4}}$$

where D is the outer diameter. For a series of rods below each other:

$$\alpha \approx 0.728 \left(\frac{\lambda^3 \rho^2 \Delta H g}{Z^{2/3} \eta D \Delta T} \right)^{\frac{1}{4}}$$

where Z is the number of rods.

For condensation inside a *short* horizontal tube and not too much condensate:

$$\alpha \approx 0.575 \left(\frac{\lambda^3 \rho^2 \Delta H g}{\eta d \Delta T} \right)^{\frac{1}{4}}$$

where d is the inner diameter. (If much condensate forms then it prevents good heat transport, and can even fill up the tube.)

Presence of inert gases (e.g. air) deteriorates the heat transport in a great extent because the partial pressure of the condensing component is decreased.

There are two kinds of condensation. The common kind is the one forming condensate layer, discussed above. The other one is forming condensate drops. This is preferable because more cold surface is left free on which the condensation is faster (with a factor 4 to 8). However, drop condensation is difficult to achieve; this depends on the surface material (mainly Cu and Cr alloys) and even on the condensing material. (Particularly difficult for organic vapors.)

Boiling

The film coefficient of boiling liquids depends in a special way on the temperature difference between the liquid (T) and the wall surface (T_s). Such a typical dependence is shown in **figure *****.

At small difference (e.g. up to $1 \sim 2$ K) calm boiling occurs with free convection. This region is characterized with microbubbles at the wall. Nucleate boiling belongs to higher temperature difference up to a critical point characteristic to the material. The film coefficient is highest at this point (approximately 25 C for water, and 42 C for toluene), and then it decreases with the temperature difference. The reason of this decrease is forming vapor layers (enlarged bubbles) at the wall. The heat transport through vapor is much less than through liquid. Note that the heat transport (in[W]) does not decrease but increases, even if α decreases. At very high temperatures, the film coefficient again increases because of heat radiation of the wall.

The film coefficient of boiling water at 25 C temperature difference is approximately $2 \cdot 10^4$ [W/K].

The heat transfer cannot be enhanced by mixing because the boiling makes it better.

A rough estimate for boiling water is

$$\alpha = 80 (\Delta t)^2 p^{0.6}$$

There are other estimations as well, and factors for computing α of other materials from that of water.

Thermosiphon tube. When boiling happens in vertical tubes (thermosiphon boiler, also called calandria boiler), the liquid near the bottom is just warming up, calm boiling a little higher, and nucleate boiling around the third of the tube. At about halfway greater bubbles form, and vapor layers are found even higher. these isolated layers are eventually exploded, giving rise to a labile system. This is called transient boiling region. At even higher level, the vapor islands unify to form a vapor plug, pressing a liquid layer to the wall. If the tube is too long then at the top of the tube is dry.

Subcooled boiling. Effective heating of liquid can be achieved by keeping the wall surface at about 5 C above the boiling point of the liquid. Then calm boiling happens just at the wall. The microbubbles enter the cold bulk of the liquid, condensate there, and maintain large heat flux.

Spray cooling. Condensation at atmospheric temperature (air condensers) can be enhanced by spraying just so small amount of water to the outer wall of the condenser that evaporates and takes over the heat.

6.4 Heat radiation

Absorption, radiation, emission, and color

When electromagnetic radiation (like heat) reaches a body, the energy carried by it partially absorbed, reflexed, or transmitted. Thus, if E_{in} is the incoming energy, the parts are respectively $Q_A = A E_{in}$, $Q_R = R E_{in}$, $Q_D = D E_{in}$ (D stands for 'diathermy'), where the factors satisfy $A + R + D = 1$. For solid bodies $D = 0$, but some are (partially) transparent, like glass.

The electromagnetic radiation consists of several constituent parts of different frequencies (or wavelengths). Bodies absorb and reflect the waves selectively; and this selectivity depends on the material. Color of a body is caused by this selectivity.

The bodies radiate heat. The emitted energy is proportional to the 4th degree of the absolute temperature, according to experience, and can be expressed as

$$q = \varepsilon \sigma T^4$$

where σ is the *Stephan-Boltzmann* constant

$$\sigma = 5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$$

and $0 < \varepsilon < 1$ is the material dependent *emissivity*.

Distribution of the frequency of the emitted energy also depends on temperature of the body. At high temperature the maximum is shifted towards lower wavelengths, i.e. higher frequencies (law of *Wien*: $\lambda_{\max} \cdot T = \text{constant}$). The visible colors form a narrow wavelength interval around the maximum of the distribution at 6000 K, the temperature of the Sun. Approximately 45.8% of the Sun's radiation belong to the visible colors. Contrary, radiation of an oven (below 500 K) contains mostly infrared waves, and less than its 0.01% is visible.

By definition, an 'ideal' *black body* absorbs every wavelength radiation. There are a few 'gray' materials which absorb approximately uniformly. Such materials are e.g. Ni, slate, tarred felt, oil cloth, black paper. These materials absorb infrared light approximately in the same extent as visible light.

Most bodies are, however, colored. Light colors, especially white, are caused by reflecting much percent of the visible light. Thus, for example, if a white paper sheet is exposed to sunshine then it does not warm up ($A \approx 0.10$). However, it warms up and even bursts into fire near an oven ($A \approx 0.95$). Contrary, a black paper sheet warms up in both cases ($A \approx 0.95$ for infrared, and $A \approx 0.94$ for visible radiation).

Non-conducting materials, like metal oxides, stones, wood, rubber etc. behave like white paper. Conducting materials like metals provide good reflection under 500 K, and good absorption under Sun.

Radiation transport between two facing surfaces

The *Stephan-Boltzmann* law expresses the radiation of a black body as $q_b = \sigma T^4$ so that for a real body $q = \varepsilon q_b$. If a body is in thermal equilibrium with its surroundings then it emits just as much energy as it absorbs: $Aq_b = \varepsilon q_b$; thus $A = \varepsilon$.

Effective radiation of a body consists of its emission and reflexion of the incoming radiation. Consider two equal area facing surfaces with shown in **figure *****, and suppose $D_1 = D_2 = 0$, so that $R = 1 - A$. Denote the self-radiation $\varepsilon\sigma T^4$ by E , and the effective radiation by H . Then, by $A = \varepsilon$:

$$\begin{aligned} H_1 &= E_1 + (1 - \varepsilon_1) H_2 \\ H_2 &= E_2 + (1 - \varepsilon_2) H_1 \end{aligned}$$

because they reflect each others radiation. By substituting each other:

$$\begin{aligned} H_1 &= E_1 + (1 - \varepsilon_1) [E_2 + (1 - \varepsilon_2) H_1] \\ H_2 &= E_2 + (1 - \varepsilon_2) [E_1 + (1 - \varepsilon_1) H_2] \end{aligned}$$

and this can be solved as

$$\begin{aligned} H_1 &= \frac{E_1 + (1 - \varepsilon_1) E_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \\ H_2 &= \frac{E_2 + (1 - \varepsilon_2) E_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \end{aligned}$$

The radiation transport from body 1 to body 2 is $Q = H_1 - H_2$, i.e.

$$Q = \frac{\varepsilon_2 E_1 - \varepsilon_1 E_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

Since $E_1 = \varepsilon_1 \sigma T_1^4$ and $E_2 = \varepsilon_2 \sigma T_2^4$, one may write

$$Q = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \sigma (T_1^4 - T_2^4) \equiv \varepsilon_{1,2} \sigma (T_1^4 - T_2^4)$$

where

$$\frac{1}{\varepsilon_{1,2}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

If $\varepsilon_2 = 1$ (black body) then $\varepsilon_{1,2} = \varepsilon_1$.

If $\varepsilon_1 = \varepsilon_2$ then $\varepsilon_{1,2} = \frac{\varepsilon}{2-\varepsilon}$.

Screening

Consider three equal area surfaces, i.e. two surfaces (1) and (2) and a screen (s) in between as is shown in **figure *****, with equal emissivities $\varepsilon_1 = \varepsilon_s = \varepsilon_2$, and with temperatures T_1 , T_s , and T_2 . We know from the preceding subsection that $\varepsilon_{1,2} = \frac{\varepsilon}{2-\varepsilon}$ if no screen is present. That is,

$$q_{1,2}^{\text{no screen}} = \frac{\varepsilon}{2-\varepsilon} \sigma (T_1^4 - T_2^4)$$

On the other hand, $\varepsilon_{1,s} = \varepsilon_{s,2} = \frac{\varepsilon}{2-\varepsilon}$, in the same way. It follows that

$$q_{1,s} = \frac{\varepsilon}{2-\varepsilon} \sigma (T_1^4 - T_s^4)$$

$$q_{s,2} = \frac{\varepsilon}{2-\varepsilon} \sigma (T_s^4 - T_2^4)$$

In steady state $q = q_{1,s} = q_{s,2}$, therefore

$$T_1^4 - T_s^4 = T_s^4 - T_2^4$$

so that

$$T_s^4 = \frac{1}{2} (T_1^4 + T_2^4)$$

By substitution

$$q = q_{1,s} = \frac{\varepsilon}{2-\varepsilon} \sigma \left[T_1^4 - \frac{1}{2} (T_1^4 + T_2^4) \right] = \frac{\varepsilon}{2-\varepsilon} \frac{1}{2} \sigma (T_1^4 - T_2^4)$$

that is

$$q = \frac{1}{2} q_{1,2}^{\text{no screen}}$$

Consider now several, generally n screens between the two end surfaces, as is shown in **figure *****. In the same way as above, in steady state $T_1^4 - T_{s1}^4 = T_{s1}^4 - T_{s2}^4 = \dots = T_{sn}^4 - T_2^4$ and, finally,

$$q = \frac{1}{n+1} q_{1,2}^{\text{no screen}}$$

Thus, effective insulation can be achieved by a series of screens.

6.5 Overall heat transport

Here we consider heat transport between two fluids separated by a wall, as is shown in **figure *****. At the warmer side, the heat is transported from the warm fluid 1 to the cooler, but yet warm surface of the wall by conduction and convection. The total effect is expressed as

$$q = \alpha_1 (T_1 - T_{s1})$$

where α_1 is the film coefficient on this side. Inside the wall the heat is transported by conduction only. Considering a homogeneous wall with conductivity λ and width w , the effect is described as

$$q = \frac{\lambda}{w} (T_{s1} - T_{s2})$$

At the other side, the heat is transported from the surface to the cooler fluid 2 by conduction, convection, and even radiation. For dense fluids, like liquids, radiation can be neglected. The total effect is expressed as

$$q = \alpha_2 (T_{s2} - T_2)$$

where α_2 is the film coefficient on this side.

All these three heat fluxes are equal in steady state. The overall effect can be expressed as

$$q = U (T_1 - T_2)$$

where U is **overall heat transfer coefficient**:

$$\frac{1}{U} = \frac{1}{\alpha_1} + \frac{w}{\lambda} + \frac{1}{\alpha_2}$$

(resistances are added). In practice, a modified coefficient is applied as

$$\frac{1}{U'} = \left(\frac{1}{\alpha_1} + \beta_1 \right) + \frac{w}{\lambda} + \left(\frac{1}{\alpha_2} + \beta_2 \right)$$

where β is the resistance due to fouling (deposits, dirtiness, fouling as well), and called *fouling factor*.

Informative film coefficients are shown in the following table.

	α [W/m ² K]
Gases	1 to 60
Superheated steam	30 to 120
Liquid, natural convection	60 to 80
Liquid, forced convection, laminar	120 to 350
Liquid, forced convection, turbulent	350 to 2,500
Boiling liquid, calm	1,700 to 7,000
Boiling liquid, nucleate	7,000 to 50,000
Condensing organic vapor	1,200 to 2,500
Condensing steam, film condensation	6,000 to 14,000 but down to 600 or 300 if fouling!
Condensing steam, drop condensation	30,000 to 40,000

Chapter 7

Heat exchange arrangements

7.1 Batch arrangements

Twice constant temperatures

This is the simplest arrangements because both temperatures are constant in time. It is also called 'both infinite heat capacities' arrangements, but this term is a little bit misleading. Such an arrangement is evaporative boiling a finite amount of liquid by application of condensing vapor (steam) stream as heating medium, as is shown in **figure *****. The amount of liquid decreases during the process; this is why it is a batch one. In this case the original equation can be used directly to compute the heat transport: $q = U (T_{\text{steam}} - T_{\text{boiling}})$.

One constant temperature

Here one of the fluids can be considered of infinite capacity, the other one has finite capacity, and perfect mixing is assumed.

One example is warming up a finite amount of liquid or gas by application of condensing vapor (steam) stream as heating medium, as is shown in **figure *****. Temperature of the fluid increases monotonically in time from the initial temperature T_0 and asymptotically approaches T_{steam} .

Another example is cooling down a finite amount of liquid or gas by application of air cooling in wind with constant ambient temperature. (Thus, the heat taken over by the air is carried away, and does not increase the temperature of the fresh air.) Temperature of the fluid decreases monotonically in time from the initial temperature T_0 and asymptotically approaches $T_{\text{fresh air}}$.

From the viewpoint of the finite amount fluid, discussion of the process is symmetric in the two cases. Here we deal with the cooling. The heat transported in a differential time period is

$$dQ = U A (T - T_c) dt$$

where T_c is the temperature of the cooling medium. On the other hand, the heat lost by the finite amount fluid because of decreasing its temperature is

$$dQ = -m c_p dT \equiv -\varphi dT$$

where m is the mass of the fluid, and φ is its heat capacity. These two values must be equal:

$$\begin{aligned} -\varphi dT &= U A (T - T_c) dt \\ \frac{dT}{T - T_c} &= -\frac{U A}{\varphi} dt \\ \ln \frac{\Delta}{\Delta_0} &\equiv \ln \frac{T - T_c}{T_0 - T_c} = -\frac{U A}{\varphi} t \end{aligned}$$

$$\boxed{\frac{\Delta}{\Delta_0} = e^{-\frac{U A}{\varphi} t}}$$

$$T = T_c + \Delta_0 e^{-\frac{U A}{\varphi} t}$$

$$T = T_0 - (1 - \Delta_0) e^{-\frac{U A}{\varphi} t}$$

as is shown in **figure *****.

The heat transported during time t is computed as

$$Q = U A \int_0^t (T(\tau) - T_c) d\tau$$

It would be much better, however, to compute it as

$$Q = U A (\Delta T)_{\text{mean}} t$$

By rearrangement, substitution, and integration

$$(\Delta T)_{\text{mean}} t = \frac{Q}{U A} = \int_0^t \Delta(\tau) d\tau = \Delta_0 \int_0^t e^{-\frac{U A}{\varphi} \tau} d\tau = -\Delta_0 \frac{\varphi}{U A} \left[e^{-\frac{U A}{\varphi} t} - 1 \right]$$

$$(\Delta T)_{\text{mean}} = \frac{\Delta_0 \varphi}{U A t} \left[1 - e^{-\frac{U A}{\varphi} t} \right] = \frac{\Delta_0 \varphi}{U A t} \left[1 - \frac{\Delta(t)}{\Delta_0} \right] = \Delta_0 \ln \frac{\Delta_0}{\Delta} \left[\frac{\Delta - \Delta_0}{\Delta_0} \right]$$

$$\boxed{(\Delta T)_{\text{mean}} = (\Delta T)_{\text{ln}} = \frac{\Delta_0 - \Delta}{\ln \frac{\Delta_0}{\Delta}} = \frac{\Delta - \Delta_0}{\ln \frac{\Delta}{\Delta_0}}}$$

This is the so-called **logarithmic mean**. The logarithmic mean applies for heating, as well.

Finite capacities

Such an arrangement is shown in **figure *****. The warmer fluid cools down; the cooler fluid warms up. Again, perfect mixing is assumed in both fluids. First it is worth to determine the common temperature to which both temperatures approach asymptotically at infinite time. This can be accomplished by taking into account the heat balance:

$$(T_{1,0} - T_{\infty}) \varphi_1 = (T_{\infty} - T_{2,0}) \varphi_2$$

$$T_{\infty} = \frac{\varphi_1 T_{1,0} + \varphi_2 T_{2,0}}{\varphi_1 + \varphi_2}$$

The differential heat balance can be written as cooling = warming, i.e.

$$dQ = -\varphi_1 dT_1 = \varphi_2 dT_2$$

$$dT_1 = -\frac{\varphi_2}{\varphi_1} dT_2$$

After integration from zero to some finite time:

$$T_1 - T_{1,0} = -\frac{\varphi_2}{\varphi_1} (T_2 - T_{2,0})$$

$$T_1 = T_{1,0} + \frac{\varphi_2}{\varphi_1} (T_{2,0} - T_2)$$

$$T_1 - T_2 = \frac{\varphi_1 + \varphi_2}{\varphi_1} T_2 + \frac{\varphi_1 T_{1,0} + \varphi_2 T_{2,0}}{\varphi_1}$$

Heat loss of the warmer fluid because of temperature decrease must be equal to the heat transported to the other fluid:

$$-\varphi_1 dT_1 = U A (T_1 - T_2) dt = U A \left[\frac{\varphi_1 + \varphi_2}{\varphi_2} T_1 + \frac{\varphi_1 T_{1,0} + \varphi_2 T_{2,0}}{\varphi_2} \right] dt$$

$$-\varphi_1 dT_1 = U A \frac{\varphi_1 + \varphi_2}{\varphi_2} (T_1 - T_{\infty}) dt$$

$$\frac{dT_1}{T_1 - T_{\infty}} = -U A \frac{\varphi_1 + \varphi_2}{\varphi_1 \varphi_2} t \equiv -U A R t$$

$$\ln \frac{T_1 - T_{\infty}}{T_{1,0} - T_{\infty}} = -U A R t$$

Thus

$$\boxed{T_1 - T_{\infty} = (T_{1,0} - T_{\infty}) e^{-UARt}}$$

$$\boxed{T_{\infty} - T_2 = (T_{\infty} - T_{2,0}) e^{-UARt}}$$

and after adding them

$$T_1 - T_2 = (T_{1,0} - T_{2,0}) e^{-UARt}$$

$$\Delta = \Delta_0 e^{-UARt}$$

In the same way as earlier, logarithmic mean can be used for computing the transported heat.

7.2 Continuous arrangements

Continuous processes are designed to operate in steady state. Steady state can never be achieved perfectly in practice, but the processes can be discussed by assuming the steady state.

Instead of the finite heat capacity $\varphi = m c_p$, here φ is used to denote heat capacity flow rate: $\varphi = \dot{m} c_p$ where \dot{m} is mass flow rate.

Co-current arrangement

This ideal arrangement is achieved by entering both the hot and the cold streams at the same end of a double pipe, as is shown in **figure *****. In an imaginary infinite long device the two temperatures would asymptotically approach each other, and the temperatures run along the length of the exchanger in the same shape as along time in the case of both finite capacities batch process. Instead of process time t , here pipe length L occurs in the formulas. The temperature profile as shown in **figure ***** can be converted to linear if not the length but the transported heat is taken as independent variable; this is shown in **figure *****. Because of linearity, one can write

$$\frac{d\Delta}{dQ} = \frac{\Delta_{in} - \Delta_{out}}{Q}$$

In the same time

$$dQ = U \Delta da$$

where a is the heat transport area, thus

$$\begin{aligned} \frac{d\Delta}{U \Delta da} &= \frac{\Delta_{in} - \Delta_{out}}{Q} \\ \frac{d\Delta}{\Delta} &= U \frac{\Delta_{in} - \Delta_{out}}{Q} da \\ \ln \frac{\Delta_{in}}{\Delta_{out}} &= U \frac{\Delta_{in} - \Delta_{out}}{Q} A \end{aligned}$$

so that the logarithmic mean can be used again:

$$Q = U A \frac{\Delta_{in} - \Delta_{out}}{\ln \frac{\Delta_{in}}{\Delta_{out}}}$$

Countercurrent arrangement

This ideal arrangement is achieved by entering the hot stream at one end of a double pipe, and the cold streams at the other, as is shown in **figure *****. Three different temperature profiles are shown in **figures *****; according to the relation of heat capacity flow rates: $\varphi_1 = \varphi_2$, $\varphi_1 < \varphi_2$, or $\varphi_1 > \varphi_2$. The profiles are linear in each case when measured to Q , and with same derivation presented in the case of co-currency, we obtain the already familiar equations with logarithmic mean temperature difference:

$$Q = U A \frac{\Delta_1 - \Delta_2}{\ln \frac{\Delta_1}{\Delta_2}}$$

where $\Delta_1 = T_{1,in} - T_{2,out}$ and $\Delta_2 = T_{1,out} - T_{2,in}$ represent temperature differences at both ends of the pipe. Thus, the logarithmic mean seems to play a general role in heat exchange.

Note that $T_{2,out} \geq T_{1,out}$ may occur; that is more heat can be transported with countercurrent arrangement than with co-current one.

On the other hand, $\Delta_1 = \Delta_2$ may also occur, in which case the logarithmic mean is undefined. Numerical instability may also occur if $\Delta_1 \approx \Delta_2$. When $|\Delta_1 - \Delta_2| < 5C$ then arithmetic mean ($\frac{\Delta_1 + \Delta_2}{2}$) may be used as good approximation.

Modelling

The $Nu - Re - Pr$ equations shown in the preceding chapter refer to material properties η , ρ , and c_p . At which temperature should these properties be taken is not specified. However, the temperature of a stream is not constant, unless boiling or condensation happens at constant temperature.

Versions of the modelling equations are known according at which temperature are the material properties calculated. They can be taken in the bulk, at the temperature of the solid surface, or at some average temperature. Even if viscosity and density are taken at the wall temperature, the specific heat can be taken in the bulk. The experimental exponents of Re and Pr , as well as the constant factor are different in these cases.

In either case, an average temperature is taken along the tubes. Another usual technique is dividing the long tubes into shorter sections, and computing the properties section by section.

7.3 Heat exchangers

In a heat exchanger (abbreviated as HEX) the stream flow, and their temperatures change from the inlet point to the outlet point. Thus, applying the *Prandtl-Nusselt* relations should be subjected to some care, considering at which temperature should the material properties (density, specific heat, and viscosity) taken.

Double pipe heat exchangers

Straight double pipe heat exchanger is shown in **figure *****. Such a HEX approximates co-current or counter-current depending on where the streams are fed. Several such units can be assembled to a longer unit, in a coil shape. They are sometimes assembled one above the other, forming a vertical cooling wall, and sprinkled with water.

Shell-and-tube heat exchangers

A several tubes (even hundreds) are fixed together in a bundle and closed in one shell. The tubes provide parallel channels to one of the streams (either hot or cold one), whereas the shell is used as the channel for the other stream (cold or hot one, respectively).

For warming up cold stream, or for recovery of heat in the hot stream, the hot fluid is led to the inner tube, and the cold one to the outer space because otherwise the heat loss through the outer tube wall would be greater.

Single pass exchanger. In its simplest form (**figure *****), an elongated, wide, straight shell contains uniform straight tubes, fit in place by perforated disks, so-called tube sheets. The tubes run along the constant diameter part of the shell, to the two supporting tube sheets at the ends. There are inlet / outlet nozzles at the heads of the shell, these are used for the stream flowing in the tubes. There are also inlet and outlet nozzles on the constant diameter part of the shell, near the two ends; these serve for the stream flowing in the shell. The tube sheets serve as walls, too, in the sense that they separate the shell side from the tube side.

There are special technical constructions to cope with the heat dilatation. One of the is the so-called floating head HEX (as opposed to fixed head HEX).

The tubes can be arranged in the tube sheet in different layouts. One speaks about triangular and square pitches (**figure *****), but *pitch* means the distance between the centers of two neighbouring tubes.

The many narrow tubes together form a large heat transport surface; much larger than a double pipe exchanger with the same overall cross section area for the inner tube flow could provide. Triangular pitch is denser than the squared one, but is more difficult to clean.

Baffles are usually put in the shell, perpendicular to the tubes. Baffles are like tube sheets but a part of them is cut out (**figure *****). These baffles direct the flow in the shell, making faster or slower flow, and thus modifying Re number and the transfer coefficient.

As a result of using baffles perpendicular to the tubes, the flow in the shell is no more parallel with the tube but a crossflow is achieved.

For calculating the Re number of the flow in the shell, the flow perpendicular to the tube bundle between the baffles, and parallel with the tubes when passing a baffle, are to be computed independently, and then averaged.

Multipass heat exchangers. At one end of a double pass exchanger, the head is divided to two separated parts, and the two nozzles of the tube side is situated at the two separated parts. The other head of the exchanger is closed (**figure *****). This arrangement divides the tube bundle into two sets. The tube side stream enters in one of the nozzles, passes through the tubes belonging to that part, turns in the other head, and flows back in the tubes of the other part before it leaves the first head through the other nozzle. Thus, the tube side stream flows in a half cross section (half number of tubes), and passes approximately twice a long length. The shell side is the same as in the single pass exchanger.

Four and even six tube passes can also be achieved in similar way. The main effect of this arrangement is narrowing the cross section and thus producing faster flow in the tubes (greater Re number), for the price of higher pressure drop and pumping power.

The shell can also be made double pass by applying a baffle along the tubes in the shell. A HEX with two shell passes and four tube passes (a '2-4 HEX') is shown in **figure *****.

One can play a long time with length, shell diameter, tube diameter, vertical or horizontal positioning, tube diameter, pitches, tube number, number of passes, leading a stream in the tube or the shell, baffles, and baffle pitches, in searching for proper design providing good heat transfer coefficients, low pressure drop, no vibration, and low cost.

Correction factors. The multipass exchangers are neither co-current, nor counter-current units. Their behaviour are modelled as distorted counter-current heat exchangers, but the logarithmic mean temperature difference is corrected by multiplying it with a geometric correction factor $0 < f_G < 1$:

$$Q = U A f_G \frac{\Delta_a - \Delta_b}{\ln \Delta_a - \ln \Delta_b}$$

The proper value of this correction factor depends on the arrangement and the temperature differences of the four streams, and are plotted in charts like that shown in **figure *****.

Kettle boilers. Kettle boilers are horizontal heat exchangers with a tube bundle from one end of the shell not reaching to the other end but only to a separating weir (**figure *****). The bundle fills the width of the shell at the bottom, and the upper part of the shell is empty for providing space of the forming vapor. This boiler is used at the bottom of distillation columns to perform partial reboiling of the liquid flowing down from the column. The liquid to be boiled enters the shell near the end where the heating tubes start. That part of the liquid which is not boiled leaves the shell at the other end, behind the weir. The heating medium (usually steam, sometimes heating oil) flows in the tubes. The kettle boiler has two tube passes (to and back). This is usually achieved by bent tubes (U-tubes). However, cleaning the inside of such tubes is difficult.

Thermosiphon boilers. Thermosiphon boilers are single pass vertical heat exchangers (**figure *****) with heating medium fed to the shell, and the liquid to the tubes. This exchanger is used to perform total boiling of the liquid in the tubes. No pumping of the liquid is needed because the boiling provides the liquid with driving force to suck in at the bottom and leave at the top.

Condensers. For condensing vapor streams, the vapor is usually fed to the shell of a horizontal heat exchanger. The stream enters the shell at the top, and the condensate leaves the shell at a bottom nozzle by gravity; thus, the condensate does not cover the cooling tubes' surface. However, the condenser is sometimes partially flooded (intentionally) by the condensate for controlling the cooling power.

Finned (ribbed) tubes

The heat exchange surface of the tubes in the double pipe and shell and tube heat exchangers can be extended by fins or ribs (**figure *****). This is sometimes applied when the film coefficients at the two sides of the tube are too much different. For example, steam condenses at one side (very good heat transfer), and inert gas warms up at the other side (rather small film coefficient). In this case that side with small coefficient is enlarged with fins.

Plate heat exchangers

Several corrugated plates, usually in squared form, are faced each other, forming a prism (**figure *****). If two such plates are pressed together, the corrugated surfaces together form channels, usually 1.6 to 6 mm wide. These channels constitute the space where the streams flow. Holes made near the corners of the plates serve as inlet and outlet channels; these holes lead the streams plate by plate. Each second gap between the plates forms the space to one of the streams. Thus, for example, the space between plates 1 and 2, 3 and 4, 5 and 6, etc. form the space for the hot stream, whereas the space between plates 2 and 3, 4 and 5, 6 and 7, etc. form the space for the cold stream. This system is similar to that one used in filter press.

Spiral plate heat exchangers

The corrugated plates are rolled in spiral form, and covered at the two sides (**figure *****). One of the streams enters at the center and leaves through the nozzle at the mantle; the other one enters in another nozzle at the mantle, and leaves from the center. The channels are usually 5 to 20 mm wide. For calculation the equivalent diameter is twice the width of the channel.

Impregnated (proofed) graphite heat exchangers

Graphite is impregnated with plastic to provide resistance against corrosive materials like chlorine and chlorinated chemicals. The proofing material fills in the pores, and enhances the strength (stability) and conductivity as well. Graphite heat exchangers can be tubular exchangers or plate exchangers, but they can also be formed from one block by driving transversal rows of holes through it as is shown in **figure *****.

Barometric condensers

Small pressure vapors are sometimes condensed by directly contacting them with cooling water so that the condensate and the water leave the common mixing space together through a long downcomer (descent pipe), as is shown in **figure *****. The mixing space contains trays or packing for enhancing the phase contact. The non-condensing gases are sucked by dry vacuum, and the vacuum is also joined to the descent pipe 10 m higher than the liquid pool. This way the dry vacuum cannot pull up the liquid. On the other hand, the descending liquid also forms vacuum.

Heat pipes

Heat pipes are tubes closed at both ends, air removed from them, and filled with a small amount of liquid. One end of the tube is warmed up so that the liquid evaporates and fills in the tube. The tube is cooled at the other end, so that the vapor condenses.

In its simplest form the heat pipe is a vertical device transporting heat upwards. The condensate flows down by gravity. This arrangement can be changed by setting porous material to the inner wall of the tube. In that case these pores transport the condensate back to the cold part of the tube, and the tube can be used in any direction (**figure *****).

Boiling and condensation provide very good heat transport. Heat pipes are used in temperature control and cooling electric devices. If heat exchange has to be achieved between streams that should be prevented from contacting, there is always danger that corrosion of the heat transport device gives rise to leakage and direct contact. In such situations heat pipes can be used to transport heat between two spaces separated by double wall.

Chapter 8

Evaporation

8.1 The evaporation process

Evaporation is a process that removes a part of the volatile liquid solvent of a solution of non-volatile material (usually solid material solved in liquid). The aim of this process is concentrating the solid, at most up to its saturation (just before precipitation).

Boiling point of the solution depends of the pressure and the concentration as well. At a given pressure, the boiling point increases with the concentration (boiling point rise). For example, boiling point of atmospheric aqueous solutions is higher than 100 C.

Evaporation is also performed in order to crystallize (precipitate) the solid, but in this case the precipitation is achieved by cooling the saturated solution.

The most common solvent is water. However, when water is evaporated, the forming steam will be called vapor (not steam) in order to prevent mismatching this process steam with the utility steam usually applied for heating up the evaporator.

Unless electric heating is applied, there are five connections of an evaporator (**figure *****): (1) feed (dilute solution), (2) product (concentrated solution), (3) vapor or condensate (the removed part of the solvent), (4) steam (heating medium inlet), and (5) return (waste) water (heating medium outlet).

Heat and material balance. For calculating the heat power necessary to operate the evaporator, the specifications and heat balance are to be taken into account. Usually the feed flow rate, feed temperature, feed and product concentration, and the pressure are specified, as well as the heating steam data.

The net heat power absorbed by the evaporation process can be calculated by

$$Q_{net} = L_0 \cdot c_p \cdot (T_{in} - T_b) + V \cdot \Delta H$$

where L_0 is the feed flow rate, c_p is the feed's specific heat, T_{in} is the feed temperature, T_b is the boiling point of the product (not the feed) at the pressure of the

evaporation space, V is the flow rate of the vapor formed in the process, and ΔH is the specific vaporization heat of the solvent at boiling point T_b . The first part is the heat necessary to warm up the feed to boiling point; the second part is the heat necessary to evaporate a part of the solvent.

Naturally

$$L_0 = V + L$$

where L is the product flow rate. The usual measure of concentration is mass fraction x . Since no solute is evaporated,

$$L_0 \cdot x_0 = L \cdot x$$

where x_0 is the concentration in the feed, and x is concentration in the product. Thus,

$$L = L_0 \frac{x_0}{x}$$

$$V = L_0 \left(1 - \frac{x_0}{x}\right)$$

The heat balance can be written as

$$S \cdot H_S + L_0 \cdot h_0 = S \cdot h_W + L \cdot h + V \cdot H_V + Q_L$$

where S is the flow rate of the steam (and of the return water), H_S is the steam's specific enthalpy, h_W is the specific enthalpy of the return water, h_0 is the specific enthalpy of the feed, h is that of the product, H_V is specific enthalpy of the vapor, and Q_L is heat loss. Thus, the steam to be used is

$$S = \frac{L \cdot h + V \cdot H_V + Q_L}{H_S - h_W}$$

The steam enthalpy depends on the steam's pressure and temperature (usually superheated pressure is available). The enthalpy of the return water depends on the pressure in the heating jacket of the device (because its temperature is the water's boiling point at that pressure). These data can be determined with steam table.

The temperature in the evaporator can be approximately determined by calculating boiling point rise, but there are charts showing the boiling points as function of the concentration.

The enthalpies of the solution are plotted in charts (e.g Merkel plots or Mollier plots) in function of the concentration. Isobar and isotherm lines are usually also provided in these charts.

The vapor formed in the process is superheated because, due to the boiling point rise, its temperature is higher than the dew point (boiling point) of the pure solvent at the actual pressure. However, the vapor loses its extra energy by contacting with the wall of the evaporator, and the vapor leaving the outlet nozzle

can be considered as saturated vapor. For calculating the heat exchanger area of the heating in the evaporator, approximately 10 to 20 % extra heat is to be taken into account for this effect. Thus, for calculating the heat exchanger, approximately $Q = 1.15 \cdot Q_{net}$ is to be considered.

The superheating is just one source of heat loss; there are other losses through the wall, although the evaporators are usually well insulated. Approximately 20 to 50 % of the used heat power is lost.

Heat exchanger. For determining the necessary heat transfer area A , the usual

$$Q = U \cdot A \cdot \Delta T$$

is applied. However, there are different U values in the literature depending on how ΔT is defined. There are at least three different ways how the temperature in the evaporator is measured. First of all, the temperature situated in the vapor space of the evaporator shows the boiling point of the pure solvent at the given pressure. Second, the boiling point can be determined according to the boiling point rise. This temperature could be measured just at the interface between the liquid and the vapor, i.e. just at the surface level of the liquid. The vapor's temperature is the same at this point, but its temperature quickly decreases to the saturation temperature at higher points. Third, the temperature is higher deep in the liquid because of the hydrostatic pressure rise. Thus, when specifying a heat transfer coefficient, where the temperature is measured should also be specified.

Preheating. The heat exchanger applied in the evaporator is usually more expensive than applicable to the feed because the more concentrated solution's viscosity is also greater, and the temperature is higher and, additionally, a special arrangement is used for the evaporator. This, it is usually more economical to heat near to its boiling point before entering the evaporator. If the pressure of the feed stream is higher than that in the evaporator body (as is usual), the feed temperature can be higher than its boiling point at the pressure inside.

8.2 Evaporators

The heat exchanger that transports the heat for evaporating a part of the solvent can be set inside the evaporator or outside. Old evaporators have internal boiling devices, but later constructions apply external boilers.

Natural circulation of the liquid is applied if the viscosity is small; but the circulation is enforced by pump when viscous liquid is evaporated.

Robert evaporator

This is an old type evaporator, developed for sugar factories, with internal heat exchanger and natural circulation, as is shown in **figure *****. A vertical thermosiphon-

type boiler is built in the vessel. The dilute solution is fed above the boiler. The unboiled liquid descends through the wide central channel, and the liquid boils and ascends in the narrower riser tubes. The steam condenses around the many tubes, but the liquid is cooler in the wide central channel; this effect provides the driving force for natural circulation. Air carried in the shell by the steam is let up to the vapor space. The vapor leaves through a nozzle at the top, after passing drop settlers.

Evaporators with external boiler

Some of such constructions are shown in **figure *****. The boiler tube's bundle is separated from the liquid holder space in each case. Pumping is shown in figures d and e. Plate heat exchanger is applied in figure e. Descent flow evaporator is shown in figure f. Here the boiling liquid flows downward in the tubes; this is used for heat sensitive materials because very short residence time can be achieved.

Fast evaporators

For evaporating very sensitive materials like milk or pharmaceuticals, **film evaporators** are developed. After preheating, the dilute solution is fed to the top of a vertical cylinder, and flows down as a narrow layer (0.2 to 0.6 mm) on the wall. This narrow film is maintained by scraping arms fast turned by a motor. The cylinder is heated from outside, and the volatile solvent evaporates without nucleate boiling.

For evaporating with even shorter residence time, **centrifugal evaporators** are developed. One of them is shown in **figure *****. The dilute solution is fed to the inner part of the fast turning cone and it forms a 0.1 mm wide layer with a residence time shorter than 1 s. The cone is heated from outside with steam. The dense solution is collected from the corner part of the cone. The vapor leaves in the center chimney.

Plate evaporators

Plate evaporators are essentially plate heat exchangers of condenser-boiler type. The heating steam is fed in a channel at a top corner; the return water is led out through a bottom corner. The concentrated solution and the vapor leaves the evaporator at the same hole, and separated in a subsequent unit.

8.3 Multiple effect evaporation

The vapor formed in the evaporation process is usually condensed in a heat exchanger, using cooling water. The condensation heat of the vapor can, however, also be utilized in another evaporator, as is shown in **figure *****. In this case, the vapor is applied instead of heating steam. This arrangement is usually applied if the solvent is water.

By condensing 1 kg steam, approximately 1 kg vapor can, in principle, be formed. The vapor of the second evaporator can also be used as heating medium in a third evaporator, and again approximately 1 kg vapor could be formed there if no heat loss had been. However, there are heat losses. For forming 1 kg vapor, approximately 1.1 kg steam is needed in a single evaporator, 0.57 kg in a double effect evaporator, 0.4 kg in a triple effect evaporator, etc.

The pressure of the vapor formed in the second unit is lower than that of the steam. The pressure of the vapor formed in the third unit is even lower. Thus, the saturation temperature decreases along the series of steam, vapor₁, vapor₂ etc. For achieving multiple effect, the last unit should usually be taken under vacuum.

There are co-current and counter-current arrangements. Counter-current (figure ***) provides good energetic efficiency. In this case the dilute solution enters in the unit from which the last charge of vapor is removed and then condensed, and the concentrated solution leaves the unit to which the fresh steam is fed. Thus, the hottest steam (vapor) is applied to the unit where the boiling point is highest, and mild vapor can be utilized for evaporating the dilute solution in the first unit, with low boiling point. Although this can be used for concentrating inorganic solutions, co-current (figure ***) is preferred for concentrating organic, heat sensitive materials. In such cases the concentrated solution is better boiled under vacuum, providing lower boiling temperature.

8.4 Expansion evaporation

The dilute solution is pressurized and heat up just under its boiling point at the elevated pressure before it is fed to an evaporator under lower pressure. The solution's boiling point under this lower pressure is higher than its actual temperature, and vaporization occurs without extra heating. This is called expansion evaporation. This construction is always applied with counter-current multiple effect arrangement, and frequently used for desalting sea water.

8.5 Vapor compression evaporation

The condensation heat of the vapor can also be utilized in the same unit where it is formed if the vapor is compressed. Compression of the vapor increases the boiling (dew) point of the material, and thus it is condensed at higher temperature than the boiling point of the solution. In this case no fresh steam is needed for evaporating the liquid. A sketch of such a unit is shown in figure ***. The energy consumption of this process is approximately 20 to 30 % of the original one. However, the cost of the electricity is usually higher than that of the steam. The vapor can also be compressed with steam jet compressor.