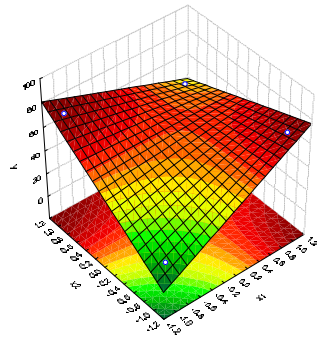


KÍSÉRLETTERVEZÉS



Mit akarunk megtudni?

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

Kétszintes faktoros tervek

1

Példa: 2⁴ teljes faktorterv elemzése

Vizsgáljuk egy kémiai reaktorban a kitermelést (%) négy faktor függvényében, ha a

- z_1 hőmérséklet 40 és 60 °C,
- z_2 reakcióidő 10 és 20 min,
- z_3 kiindulási komponens koncentrációja 45 és 65 %,
- z_4 nyomás 2 és 6 bar

KÍSÉRLETTERVEZÉS

2

Faktorok	z_1	z_2	z_3	z_4
középpont z_j^0	50	15	55	4
variációs intervallum Δz_j	10	5	10	2
felső szint $z_j^{max} (+)$	60	20	65	6
alsó szint $z_j^{min} (-)$	40	10	45	2

i	Természetes egységekben				A transzformált faktorok					y
	z_1	z_2	z_3	z_4	x_0	x_1	x_2	x_3	x_4	%
1	40	10	45	2	+	-	-	-	-	60.4
2	60	10	45	2	+	+	-	-	-	75.9
3	40	20	45	2	+	-	+	-	-	79.8
4	60	20	45	2	+	+	+	-	-	86.0
5	40	10	65	2	+	-	-	+	-	64.9
6	60	10	65	2	+	+	-	+	-	80.9
7	40	20	65	2	+	-	+	+	-	86.4
8	60	20	65	2	+	+	+	+	-	91.6
9	40	10	45	6	+	-	-	-	+	59.6
10	60	10	45	6	+	+	-	-	+	77.0
11	40	20	45	6	+	-	+	-	+	83.1
12	60	20	45	6	+	+	+	-	+	85.0
13	40	10	65	6	+	-	-	+	+	65.0
14	60	10	65	6	+	+	-	+	+	79.3
15	40	20	65	6	+	-	+	+	+	88.7
16	60	20	65	6	+	+	+	+	+	91.1

$$\hat{Y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 +$$

$$+ b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 +$$

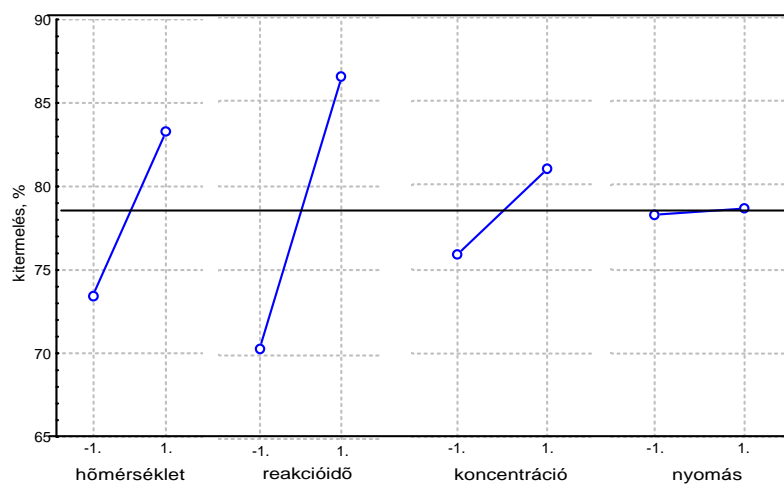
$$+ b_{123}x_1x_2x_3 + b_{124}x_1x_2x_4 + b_{134}x_1x_3x_4 + b_{234}x_2x_3x_4 + b_{1234}x_1x_2x_3x_4$$

KÍSÉRLETTERVEZÉS 5

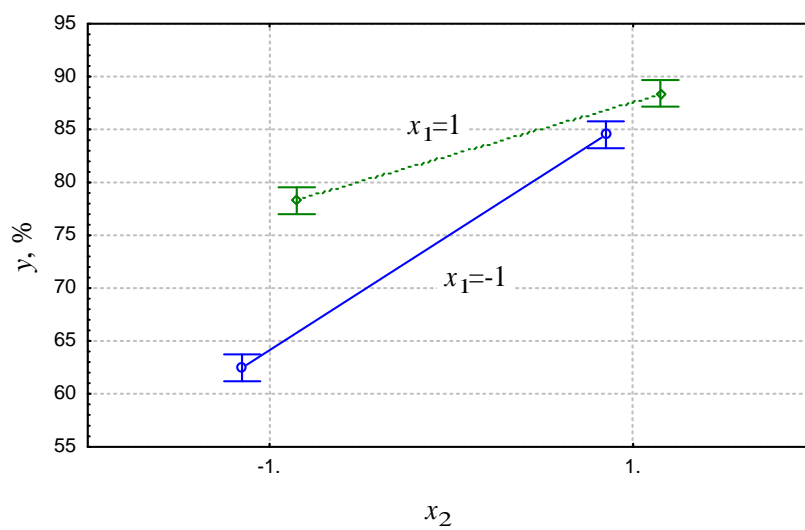
<i>i</i>	x_0	x_1	x_2	x_3	x_4	x_1x_2	x_1x_3	x_1x_4	x_2x_3	x_2x_4	x_3x_4	$x_1x_2x_3$	$x_1x_2x_4$	$x_1x_3x_4$	$x_2x_3x_4$	$x_1x_2x_3x_4$	<i>y</i>
1	+	-	-	-	-	+	+	+	+	+	+	-	-	-	-	+	60.4
2	+	+	-	-	-	-	-	-	+	+	+	+	+	+	-	-	75.9
3	+	-	+	-	-	-	+	+	-	-	+	+	+	-	+	-	79.8
4	+	+	+	-	-	+	-	-	-	-	+	-	-	+	+	+	86.0
5	+	-	-	+	-	+	-	+	-	+	-	+	-	+	+	-	64.9
6	+	+	-	+	-	-	+	-	-	+	-	-	+	-	+	+	80.9
7	+	-	+	+	-	-	-	-	+	-	-	-	+	+	-	+	86.4
8	+	+	+	+	-	+	+	+	+	-	-	+	-	-	-	-	91.6
9	+	-	-	-	+	+	+	-	+	-	-	-	+	+	+	-	59.6
10	+	+	-	-	+	-	-	+	+	-	-	+	-	-	+	+	77.0
11	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	83.1
12	+	+	+	-	+	+	-	+	-	+	-	-	+	-	-	-	85.0
13	+	-	-	+	+	+	-	-	-	-	+	+	+	-	-	+	65.0
14	+	+	-	+	+	-	+	+	-	-	+	-	-	+	-	-	79.3
15	+	-	+	+	+	-	-	-	+	+	+	-	-	-	+	-	88.7
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	91.1

KÍSÉRLETTERVEZÉS 6

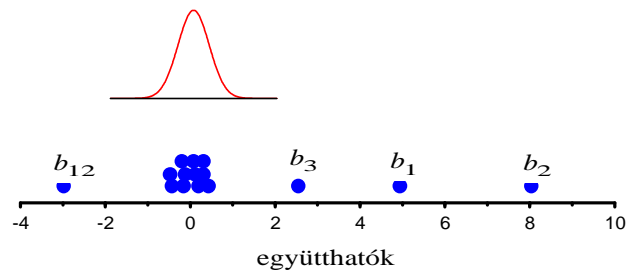
A főhatások:



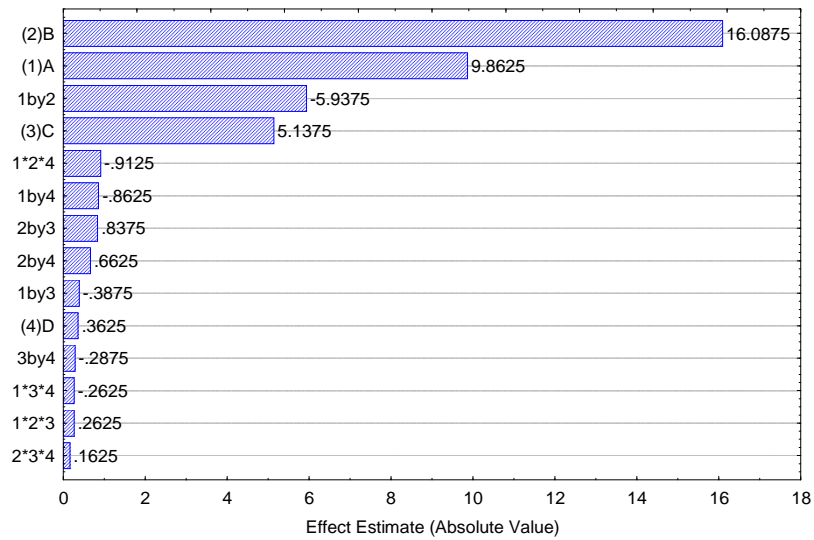
Az 1. és 2. faktor közötti kölcsönhatás:

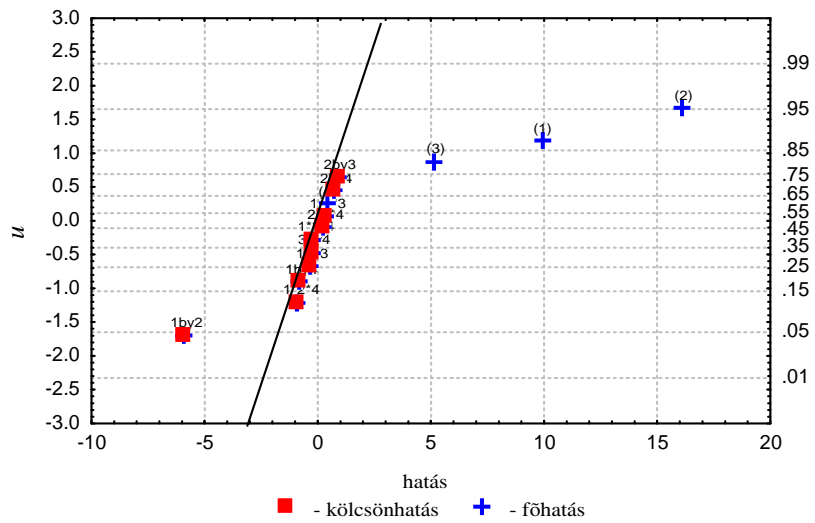


$b_0 = 78.42; b_1 = 4.93; b_2 = 8.04; b_3 = 2.57; b_4 = 0.18; b_{12} = -2.97;$
 $b_{13} = -0.19; b_{14} = -0.43; b_{23} = 0.42; b_{24} = -0.33; b_{34} = -0.14;$
 $b_{123} = 0.13; b_{124} = -0.46; b_{134} = -0.13; b_{234} = 0.08; b_{1234} = 0.32$



Pareto Chart of Effects; Variable: DV_1
 2**(4-0) design; MS Residual=1.625625
 DV: DV_1





$$\hat{Y} = 78.42 + 4.93x_1 + 8.04x_2 + 2.53x_3 - 2.97x_1x_2$$

KÍSÉRLETTERVEZÉS

11

Példa

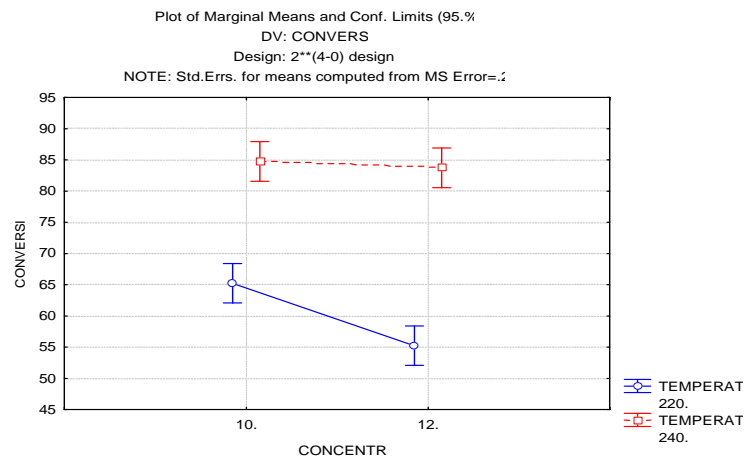
G. E. P. Box, W. G. Hunter, J. S. Hunter: Statistics for Experimenters, J. Wiley, 1978; p. 324-334

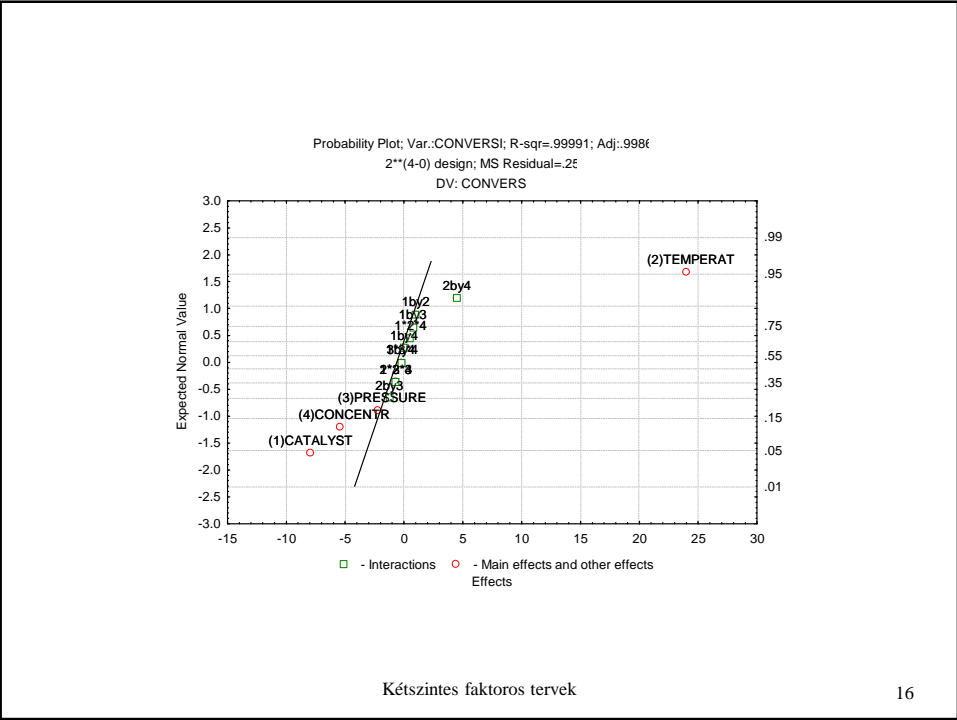
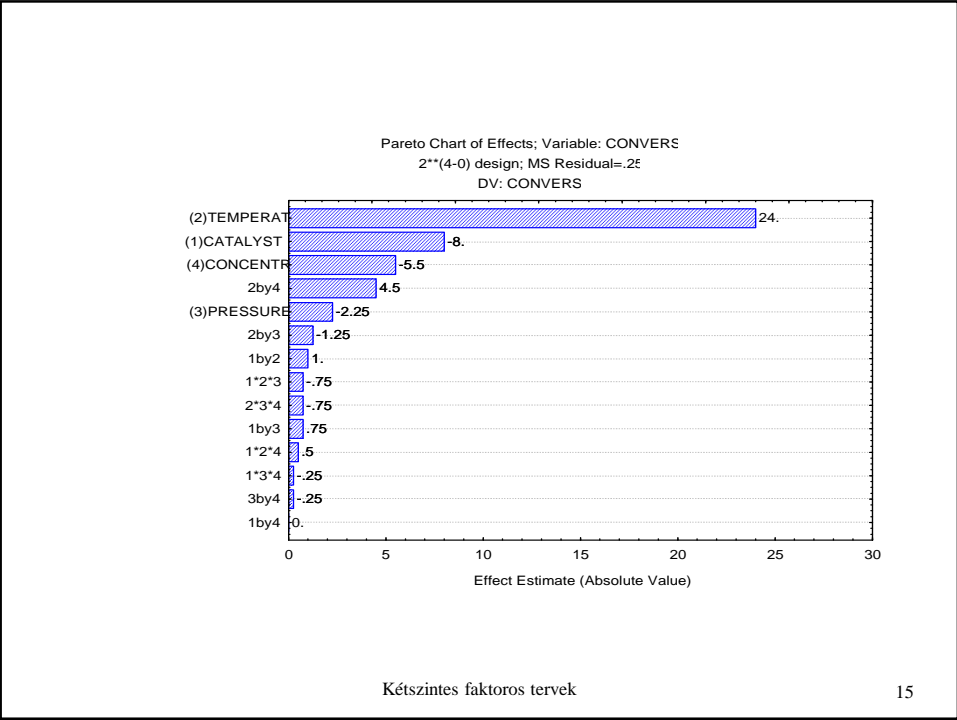
	1	2	3	4	5
	CATALYST	TEMPERAT	PRESSURE	CONCENTR	CONVERSI
1	10.000	220.000	50.000	10.000	71.000
2	15.000	220.000	50.000	10.000	61.000
3	10.000	240.000	50.000	10.000	90.000
4	15.000	240.000	50.000	10.000	82.000
5	10.000	220.000	80.000	10.000	68.000
6	15.000	220.000	80.000	10.000	61.000
7	10.000	240.000	80.000	10.000	87.000
8	15.000	240.000	80.000	10.000	80.000
9	10.000	220.000	50.000	12.000	61.000
10	15.000	220.000	50.000	12.000	50.000
11	10.000	240.000	50.000	12.000	89.000
12	15.000	240.000	50.000	12.000	83.000
13	10.000	220.000	80.000	12.000	59.000
14	15.000	220.000	80.000	12.000	51.000
15	10.000	240.000	80.000	12.000	85.000
16	15.000	240.000	80.000	12.000	78.000

Kétszintes faktoros tervek

12

Effect Estimates; Var.:CONVERSI; R-sqr=.99991; Adj:.99866 (PROCDEV.STA) 2**(4-0) design; MS Residual=.25 DV: CONVERSI							
Factor	Effect	Std.Err.	t(1)	p	-95.% Cnf.Limt	+95.% Cnf.Limt	Coeff.
Mean/Interc.	72.25000	0.125000	578.0000	0.001101	70.6617	73.83828	72.25000
(1)CATALYST	-8.00000	0.250000	-32.0000	0.019888	-11.1766	-4.82345	-4.00000
(2)TEMPERAT	24.00000	0.250000	96.0000	0.006631	20.8234	27.17655	12.00000
(3)PRESSURE	-2.25000	0.250000	-9.0000	0.070447	-5.4266	0.92655	-1.12500
(4)CONCENTR	-5.50000	0.250000	-22.0000	0.028917	-8.6766	-2.32345	-2.75000
1 by 2	1.00000	0.250000	4.0000	0.155958	-2.1766	4.17655	0.50000
1 by 3	0.75000	0.250000	3.0000	0.204833	-2.4266	3.92655	0.37500
1 by 4	0.00000	0.250000	0.0000	1.000000	-3.1766	3.17655	0.00000
2 by 3	-1.25000	0.250000	-5.0000	0.125666	-4.4266	1.92655	-0.62500
2 by 4	4.50000	0.250000	18.0000	0.035331	1.3234	7.67655	2.25000
3 by 4	-0.25000	0.250000	-1.0000	0.500000	-3.4266	2.92655	-0.12500
1*2*3	-0.75000	0.250000	-3.0000	0.204833	-3.9266	2.42655	-0.37500
1*2*4	0.50000	0.250000	2.0000	0.295167	-2.6766	3.67655	0.25000
1*3*4	-0.25000	0.250000	-1.0000	0.500000	-3.4266	2.92655	-0.12500
2*3*4	-0.75000	0.250000	-3.0000	0.204833	-3.9266	2.42655	-0.37500





Effect Estimates; Var.:CONVERSI; R-sqr=.99991; Adj:.99866 2**(4-0) design; MS Residual=.25 DV: CONVERSI	
Factor	Effect
Mean/Interc.	72.250
(1)CATALYST	-8.000
(2)TEMPERAT	24.000
(3)PRESSURE	-2.250
(4)CONCENTR	-5.500
1 by 2	1.000
1 by 3	0.750
1 by 4	0.000
2 by 3	-1.250
2 by 4	4.500
3 by 4	-0.250
1*2*3	-0.750

Effect Estimates; Var.:CONVERSI; R-sqr=.98608; Adj:.9810 2**(4-0) design; MS Residual=3.545455 DV: CONVERSI	
Factor	Effect
Mean/Interc.	72.250
(1)CATALYST	-8.000
(2)TEMPERAT	24.000
(4)CONCENTR	-5.500
2 by 4	4.500

Az ortogonalitás következménye: a becült hatások (koefficiensek) nem változnak, ha a modellt redukáljuk!
Ez a ± 1 transzformáció utáni hatásokra érvényes.

Kétszintes faktoros tervek 17

Regr. Coefficients; Var.:CONVERSI; R-sqr=.99991; Adj:.99866 2**(4-0) design; MS Residual=.25 DV: CONVERSI	
Factor	Regressn Coeff.
Mean/Interc.	729.5417
(1)CATALYST	1.1167
(2)TEMPERAT	-2.4792
(3)PRESSURE	-8.8083
(4)CONCENTR	-65.2917
1 by 2	-0.0250
1 by 3	0.2767
1 by 4	-2.0833
2 by 3	0.0358
2 by 4	0.2625
3 by 4	0.6083
1*2*3	-0.0010
1*2*4	0.0100

Regr. Coefficients; Var.:CONVERSI; R-sqr=.98608; Adj:.9810 2**(4-0) design; MS Residual=3.545455 DV: CONVERSI	
Factor	Regressn Coeff.
Mean/Interc.	415.7500
(1)CATALYST	-1.6000
(2)TEMPERAT	-1.2750
(4)CONCENTR	-54.5000
2 by 4	0.2250

Az eredeti skálán mért faktorok együtthatóira nem teljesül az ortogonalitás, azok megváltoznak, ha a modellt változtatjuk.

Kétszintes faktoros tervek 18