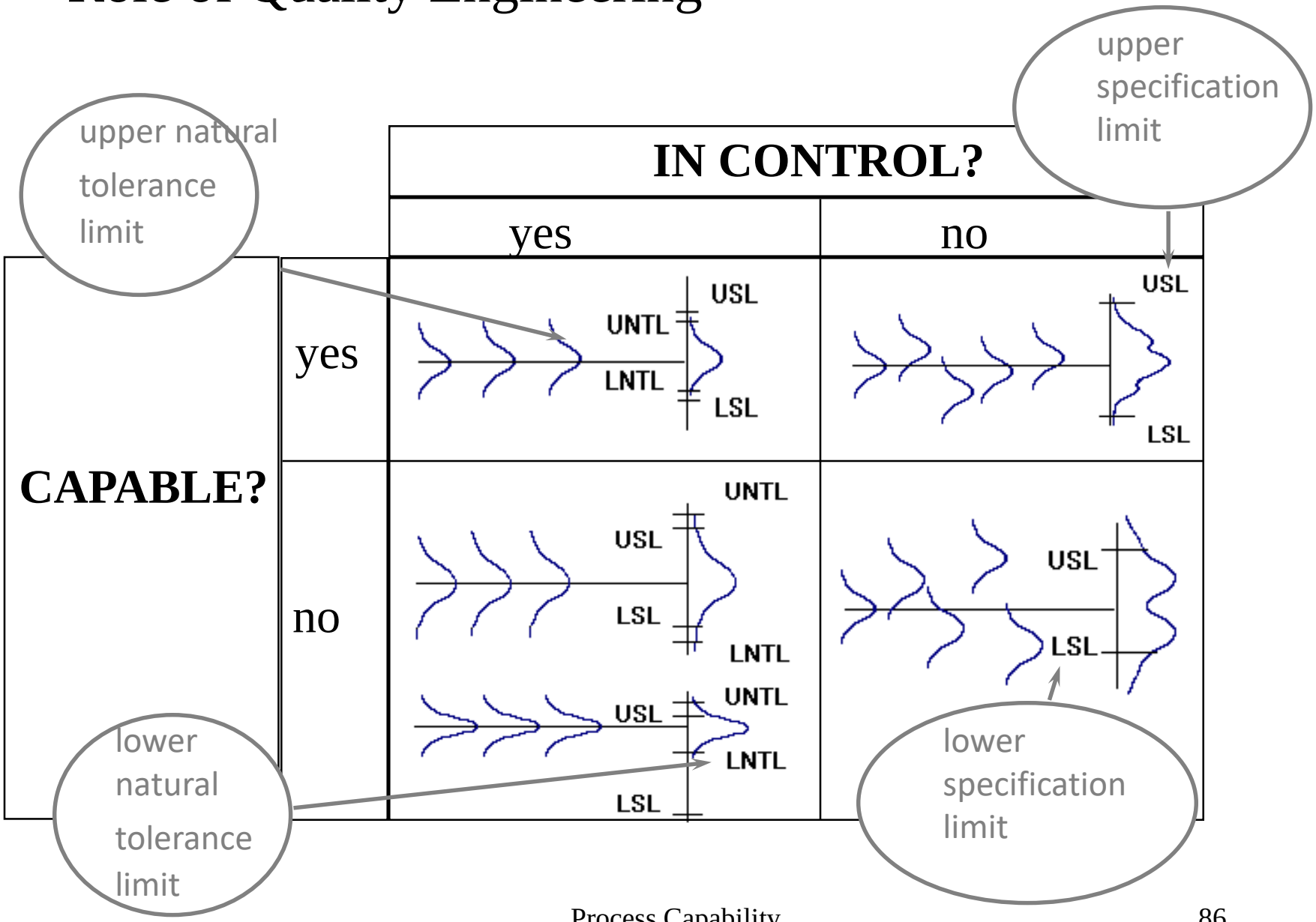
PROCESS CAPABILITY

Role of Quality Engineering



Analysis of process capability

Process capability index (Potential capability)

$$C_P = \frac{USL - LSL}{6\sigma}$$

Example 11

In a manufacturing process the expected value of a quality characteristic is μ= 250.727 unit, the standard deviation is σ= 1.286 unit. The specification is 250±5 unit.
How much is the proportion of defectives in this process?
Calculate the C_P capability index!

$$z_{\text{upper}} = \frac{USL - \mu}{\sigma} =$$

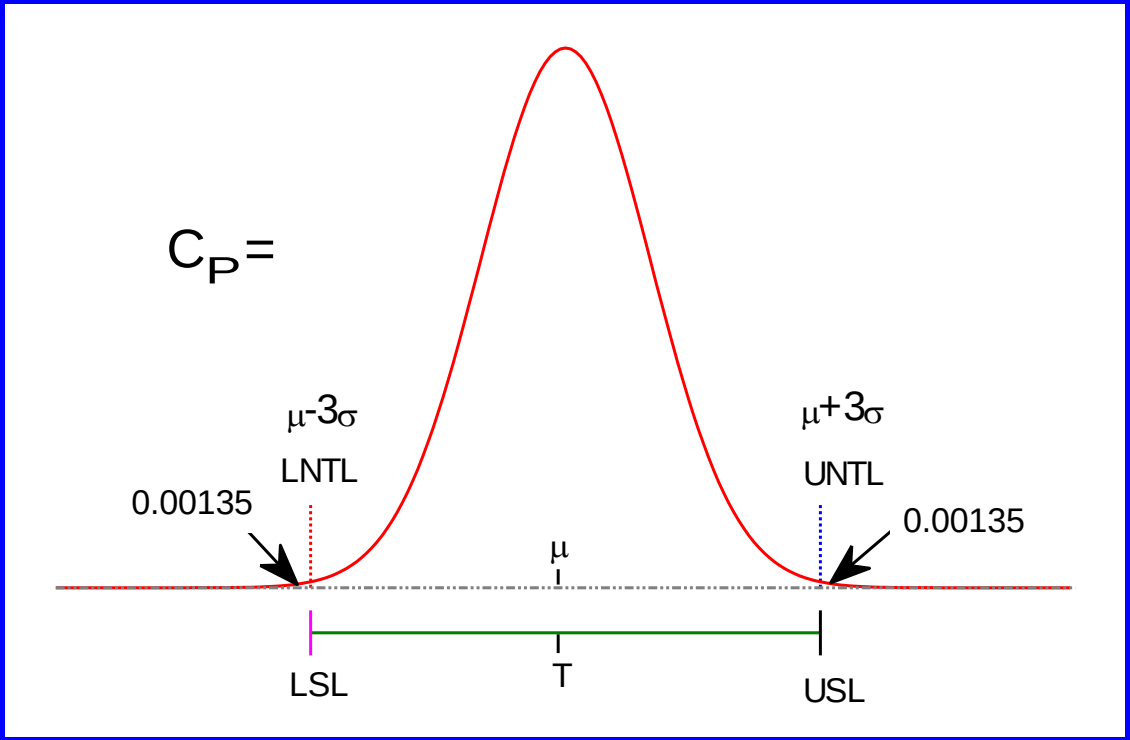
$$P(x > USL) =$$

$$z_{\text{lower}} = \frac{LSL - \mu}{\sigma} =$$

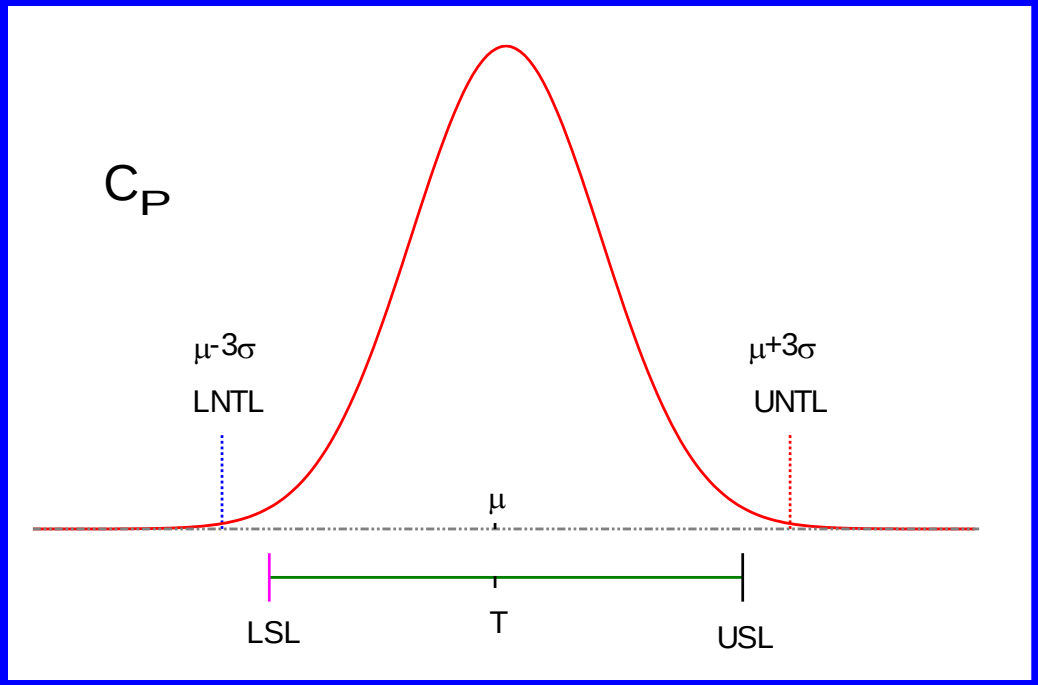
$$P(x < LSL) =$$

$$C_P = \frac{USL - LSL}{6\sigma}$$

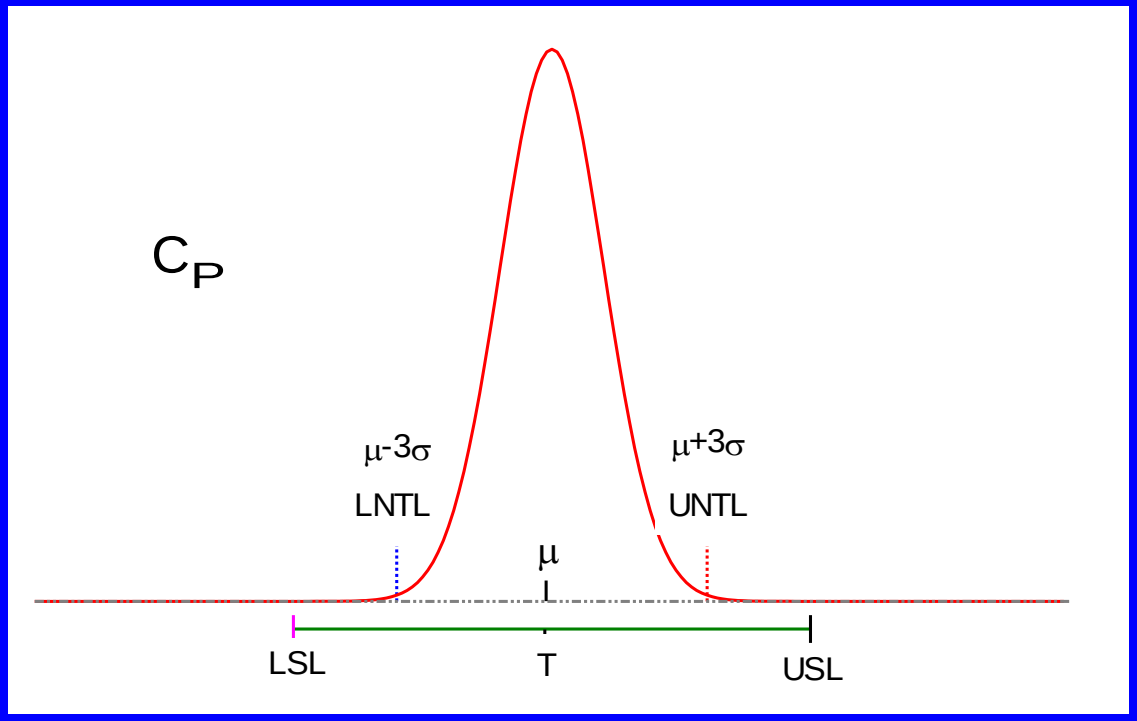
$$C_P = \frac{USL - LSL}{6\sigma}$$



$$C_P = \frac{USL - LSL}{6\sigma}$$

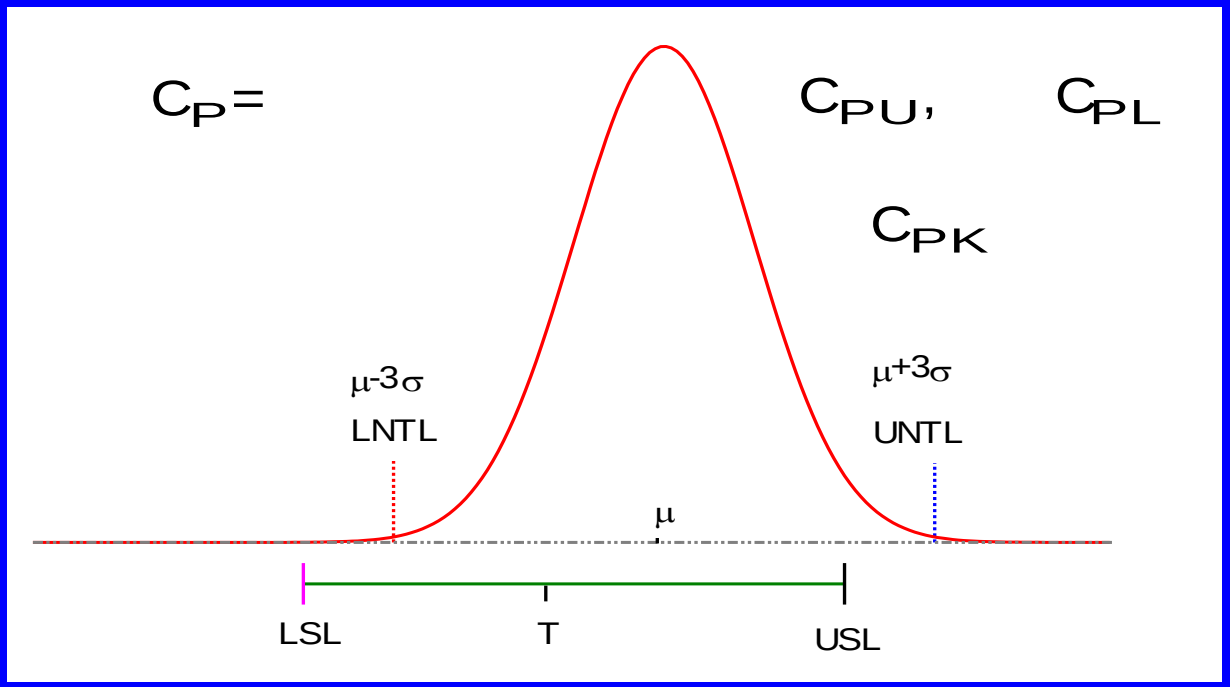


$$C_P = \frac{USL - LSL}{6\sigma}$$



Corrected indices (demonstrated capability)

$$C_{PU} = \frac{USL - \mu}{3\sigma}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma}; \qquad C_{PK} = \min\left(C_{PU}, C_{PL}\right)$$



$$C_P = \frac{USL - LSL}{6\sigma}$$

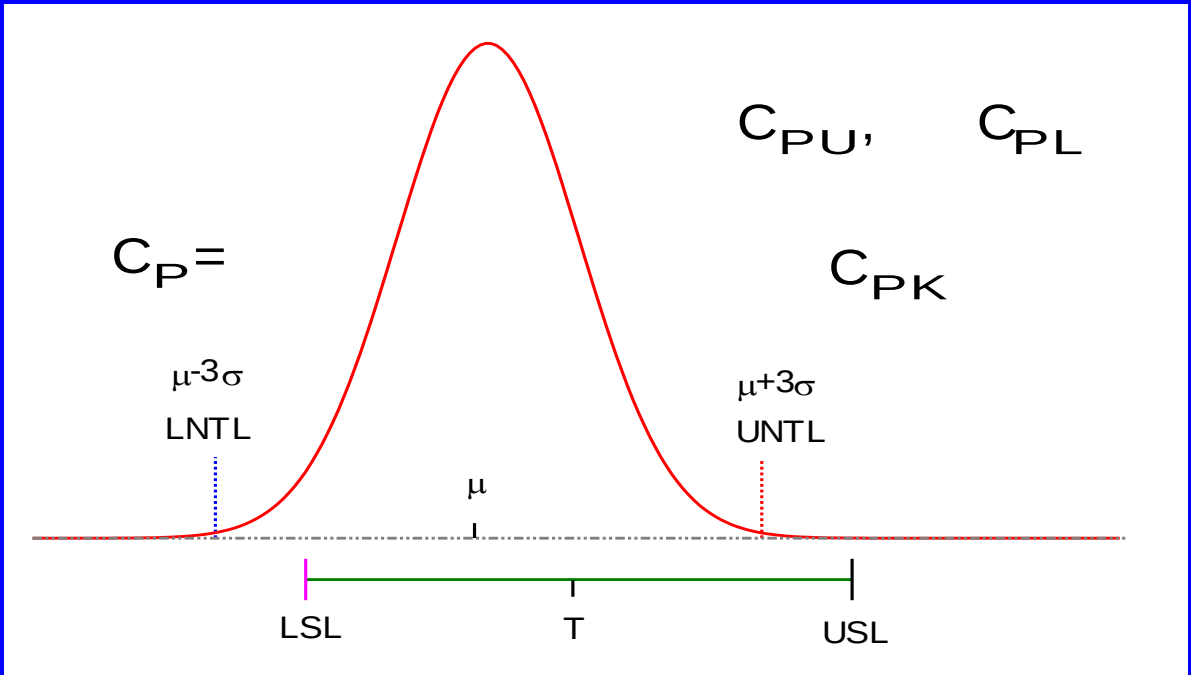
If $$T = \frac{USL + LSL}{2}$$

$$K = \frac{|\mu - T|}{\frac{USL - LSL}{2}}$$

$$C_{PK} = (1 - K)C_P$$

$$C_{PK} = \frac{\frac{USL - LSL}{2} - |\mu - T|}{3\sigma}$$

$$C_{PU} = \frac{USL - \mu}{3\sigma}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma}; \qquad C_{PK} = \min\left(C_{PU}, C_{PL}\right)$$



$$C_P = \frac{USL - LSL}{6\sigma}$$

Modified process capability index

capability index

$$C_P = \frac{USL - LSL}{6\sigma}$$

modified capability index

$$C_{Pm} = \frac{USL - LSL}{6\tau} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$MSE = E\left[(x - T)^2\right] = \tau^2$$

$$\tau^2 = \sigma^2 + (\mu - T)^2$$

related to Taguchi's quadratic loss function

Example 12

In a manufacturing process the expected value of a quality characteristic is $\mu = 250.727$ unit, the standard deviation is $\sigma = 1.049$ unit. The specification is 250 ± 5 unit.

Calculate the corrected and modified capability indices!

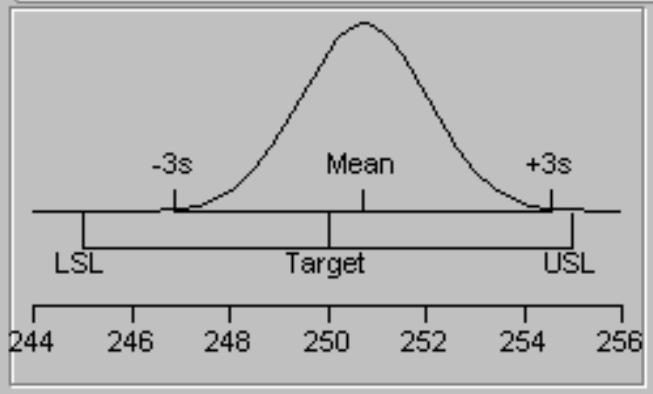
$$C_{PU} = \frac{USL - \mu}{3\sigma}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma}; \qquad C_{PK} = \min\left(C_{PU}, C_{PL}\right)$$

$$C_{Pm} = \frac{USL - LSL}{6\tau} = \frac{USL - LSL}{6\sqrt{\sigma^2 + \left(\mu - T\right)^2}}$$

YS3

Capability Analysis

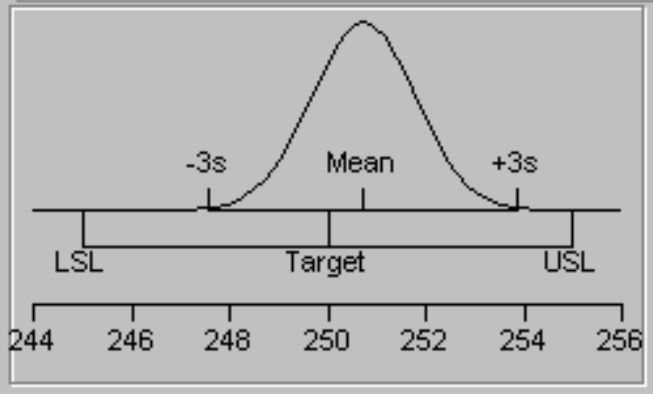
Overall, Sigma = 1.28587



Capability	Index	Lower CI	Upper CI
Pp	1.296	1.116	1.476
Ppk	1.108	0.940	1.275
CPM	1.128	0.985	1.271
PPL	1.484	1.267	1.701
PPU	1.108	0.940	1.275

Portion	Percent	PPM
Below LSL	0.0004	4.2256
Above USL	0.0445	444.5709
Total Outside	0.0449	448.7965

Subgroup variable = Sample, Sigma = 1.04962



Capability	Index	Lower CI	Upper CI
CP	1.588	1.367	1.809
CPK	1.357	1.157	1.557
CPM	1.306	1.144	1.467
CPL	1.819	1.557	2.079
CPU	1.357	1.157	1.556

Portion	Percent	PPM
Below LSL	0.0000	0.0244
Above USL	0.0023	23.3566
Total Outside	0.0023	23.3810

$\mu = 250.727$

$\sigma = 1.049$

specs: 250 ± 5

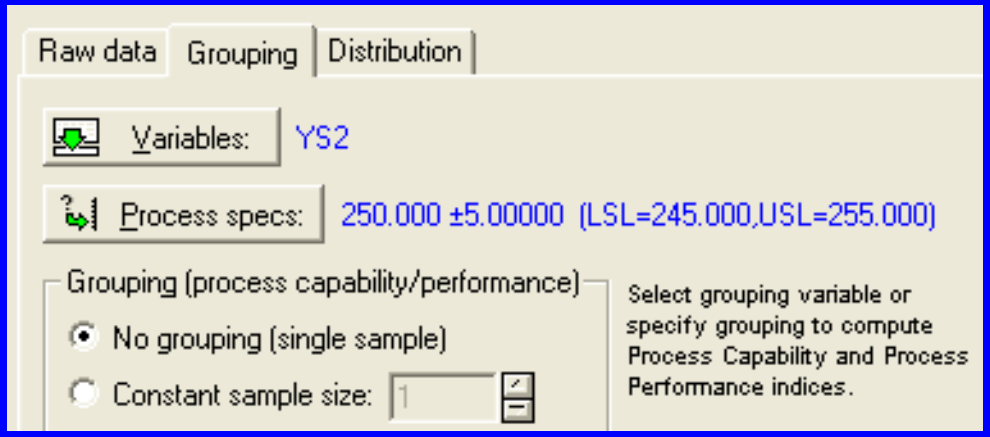
Example 3

Calculate the capability indices and the proportion beyond specs (above *USL* or below *LSL*) for YS2 data in the dataset cpdata1.mtw!
The specification is 250±5 unit.

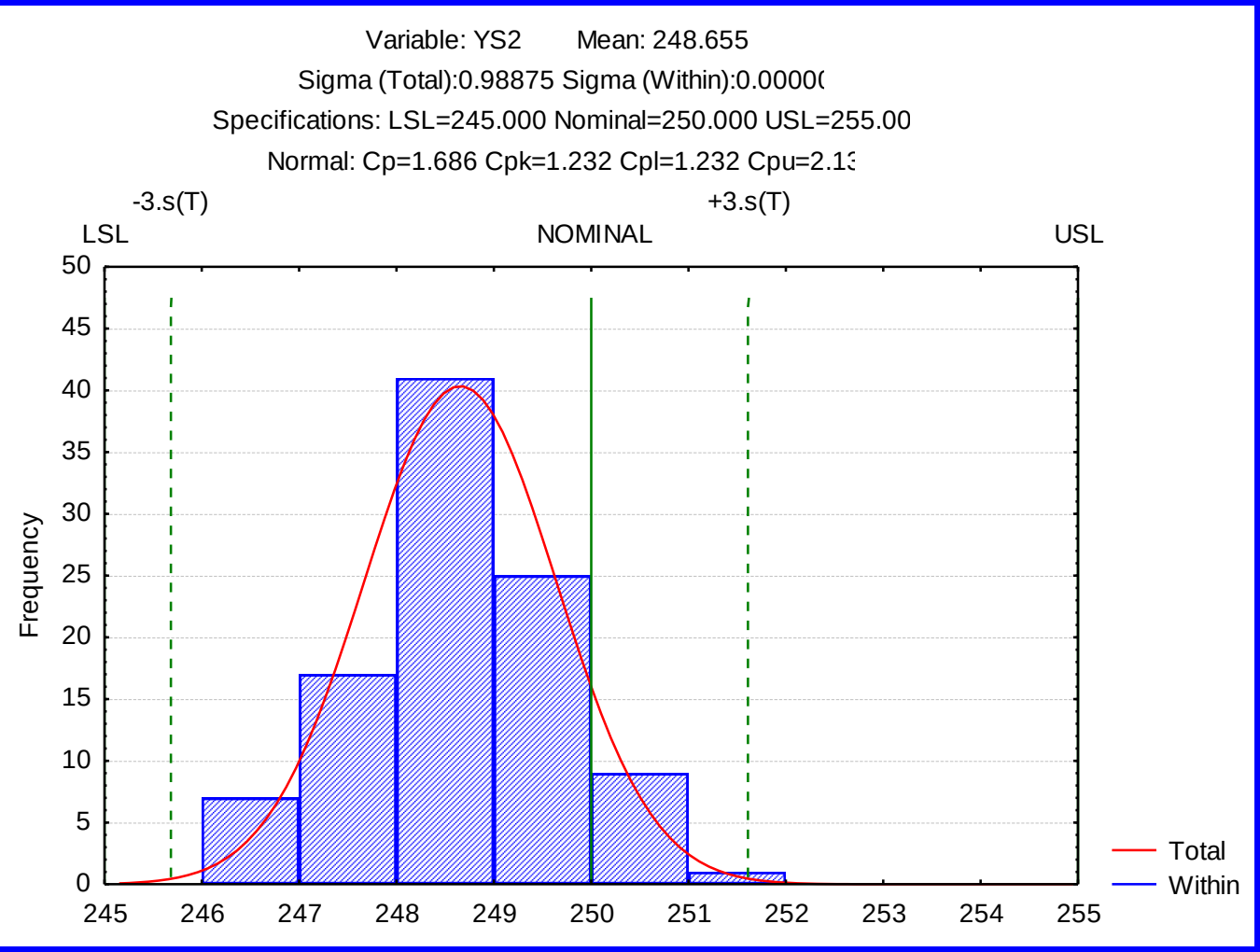
Open cpdata1.sta

Statistics>Industrial Statistics>Process Analysis
Process capability analysis

Variables: YS2
Process specs…



Summary histogram



$$s = \sqrt{\frac{\sum_{i}^{100} (x_i - \bar{x})^2}{99}}$$

μ= 248.655

σ= 0.98875

specs: 250±5

Advanced, normal

Descriptive statistics

Number beyond specs

Statistic	Descriptive Statistics (CPDATA1.STA) Variable:YS2 N = 100 Value
Mean	248.6552000(
Median	248.7050000(
25th Percentile (Q25)	248.0300000(
75th Percentile (Q75)	249.3100000(
Minimum Value	246.2600000(
Maximum Value	251.0600000(
Standard Deviation	0.9887472
Variance	0.9776211
Skewness	-0.1359641
Kurtosis	-0.0111016

	Variable: YS2, Distribution: Normal (CPDATA1.STA) Specifications: Lower=245.000 Nominal=250.000 Upper=255.00(Mean:248.66,Std.Dv:.98875			
	Observed	Percent Observed	Expected	Percent Expected
Above USL:	0	0.00	0.00000(	0.00000(
Below LSL:	0	0.00	0.010917	0.010917
Total	0	0.00	0.010917	0.010917

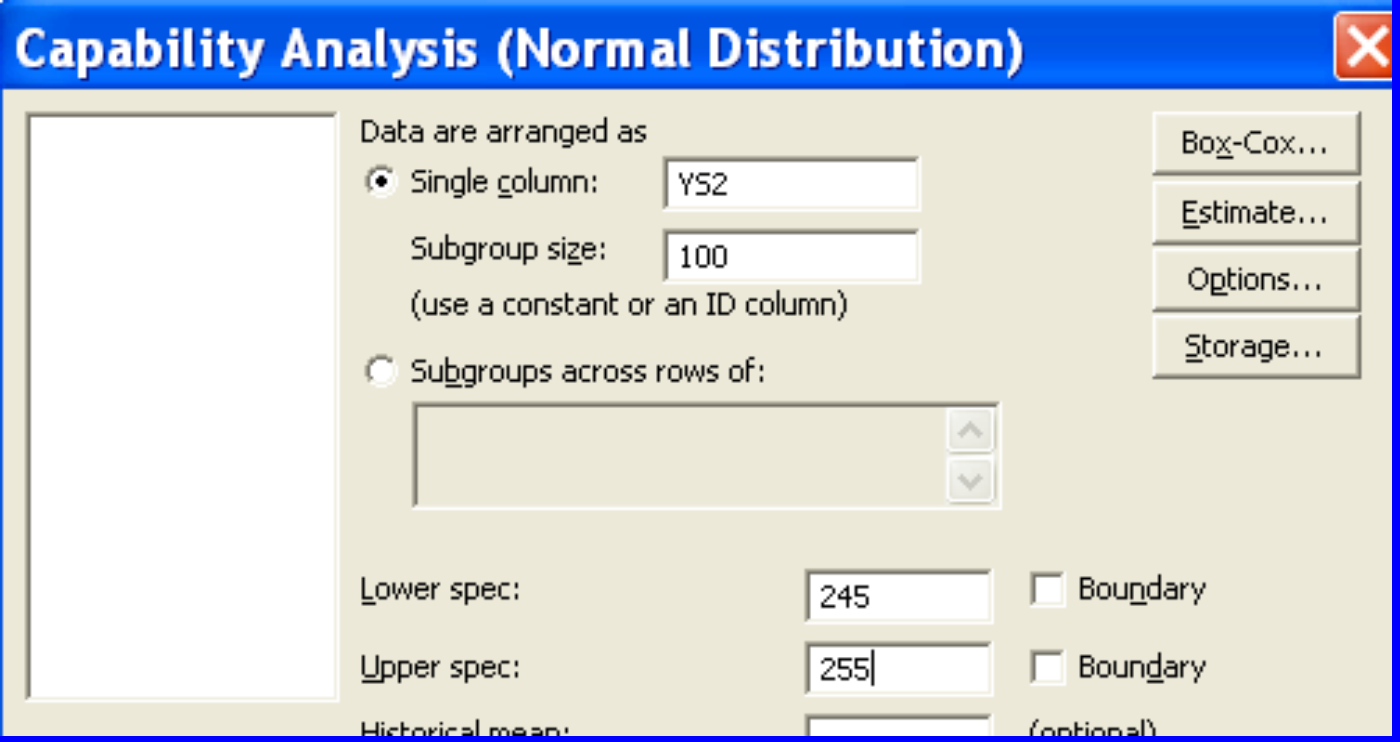
Example 3

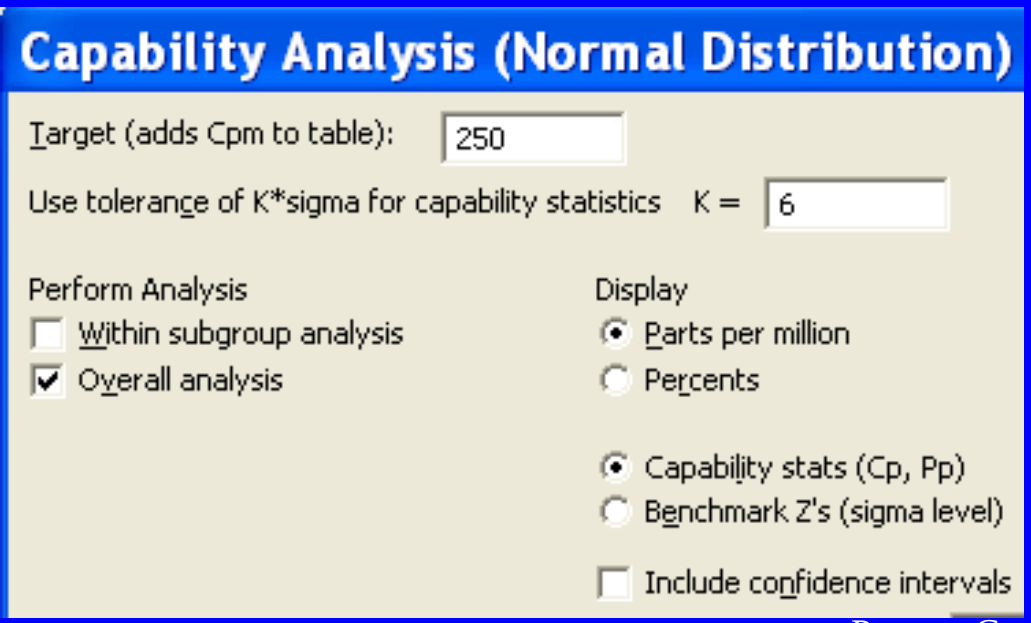
Calculate the capability indices and the proportion beyond specs (above *USL* or below *LSL*) for YS3 data in the dataset cpdata1.mtw!
The specification is 250±5 unit.

Minitab>Stat>Quality Tools>Capability Analysis>Normal
Options:

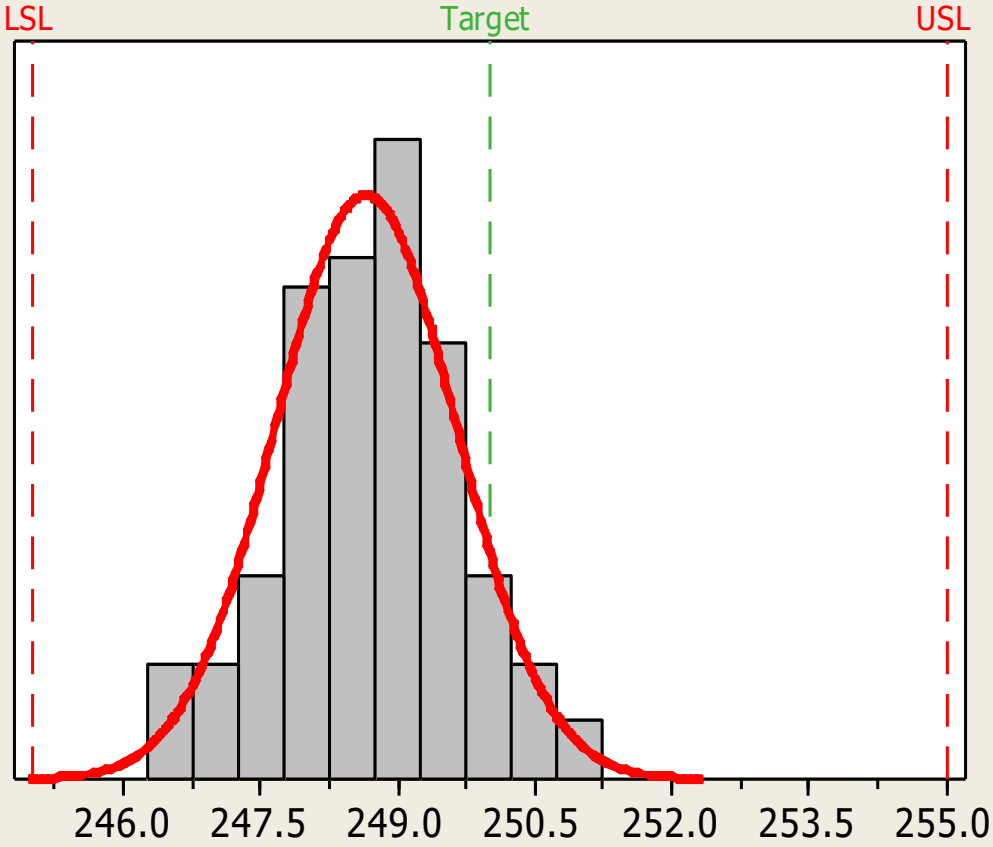
Perform Analysis: Overall analysis

Target


Capability Analysis (Normal Distribution)
Data are arranged as
Single column:
YS2
Subgroup size:
100
(use a constant or an ID column)
Subgroups across rows of:
Box-Cox...
Estimate...
Options...
Storage...
Lower spec:
245
Boundary
Upper spec:
255
Boundary


Capability Analysis (Normal Distribution)
Target (adds Cpm to table):
250
Use tolerance of K*sigma for capability statistics
K =
6
Perform Analysis
Within subgroup analysis
Overall analysis
Display
Parts per million
Percents
Capability stats (Cp, Pp)
Benchmark Z's (sigma level)
Include confidence intervals

Process Capability of YS2



Process Data	
LSL	245
Target	250
USL	255
Sample Mean	248.655
Sample N	100
StDev(Overall)	0.991247

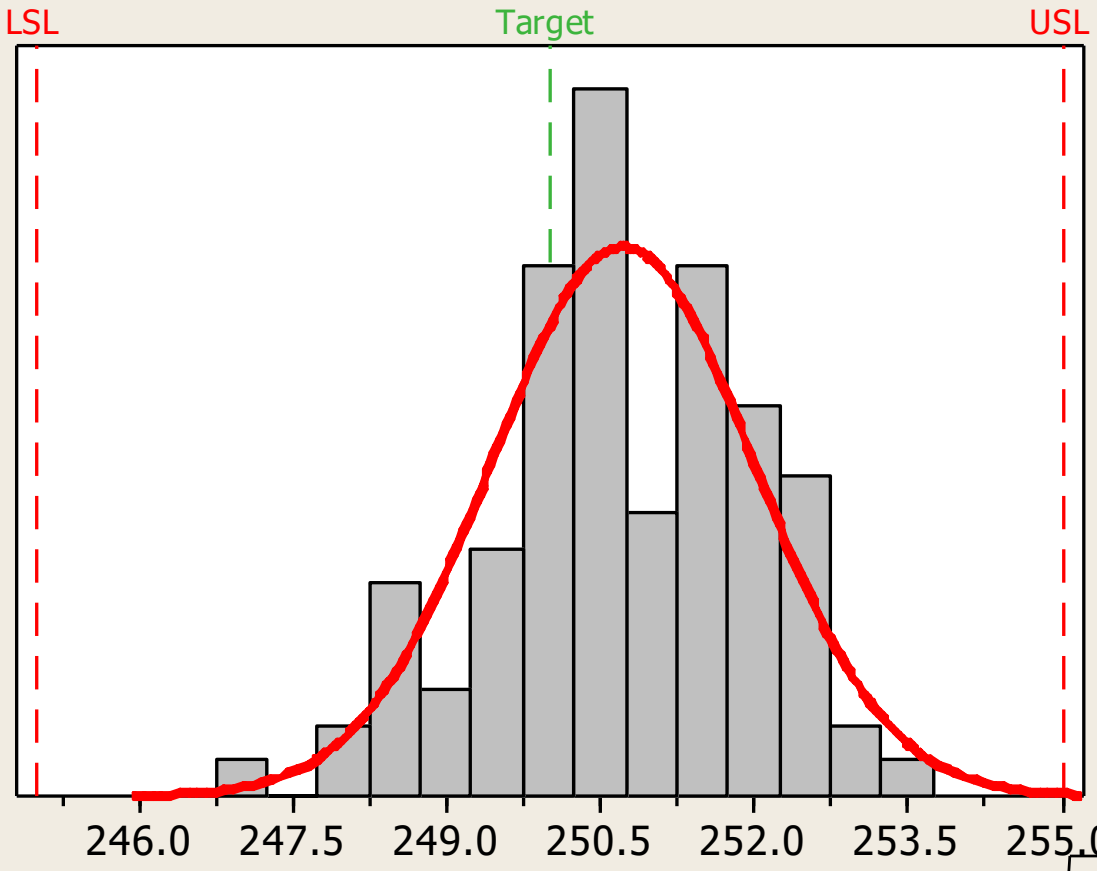
Overall Capability	
Pp	1.68
PPL	1.23
PPU	2.13
Ppk	1.23
Cpm	1.00

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Overall Performance	
PPM < LSL	113.24
PPM > USL	0.00
PPM Total	113.24

$$s_{\text{Overall}} = \sqrt{\frac{\sum_{i}^{100}\left(x_i - \bar{x}\right)^2}{99}}$$

Process Capability of YS3



Process Data	
LSL	245
Target	250
USL	255
Sample Mean	250.727
Sample N	100
StDev(Overall)	1.28587

Overall Capability	
Pp	1.30
PPL	1.48
PPU	1.11
Ppk	1.11
Cpm	1.13

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Overall Performance	
PPM < LSL	4.23
PPM > USL	444.57
PPM Total	448.79

$$s_{\text{Overall}} = \sqrt{\frac{\sum_{i}^{100}\left(x_i - \bar{x}\right)^2}{99}}$$

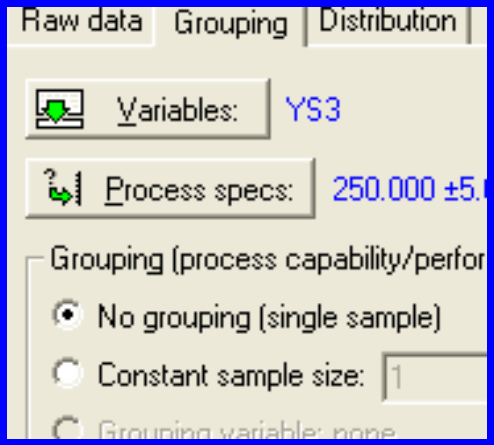
Example 3

Calculate the capability indices and the proportion beyond specs (above *USL* or below *LSL*) for YS3 data in the dataset cpdata1.sta

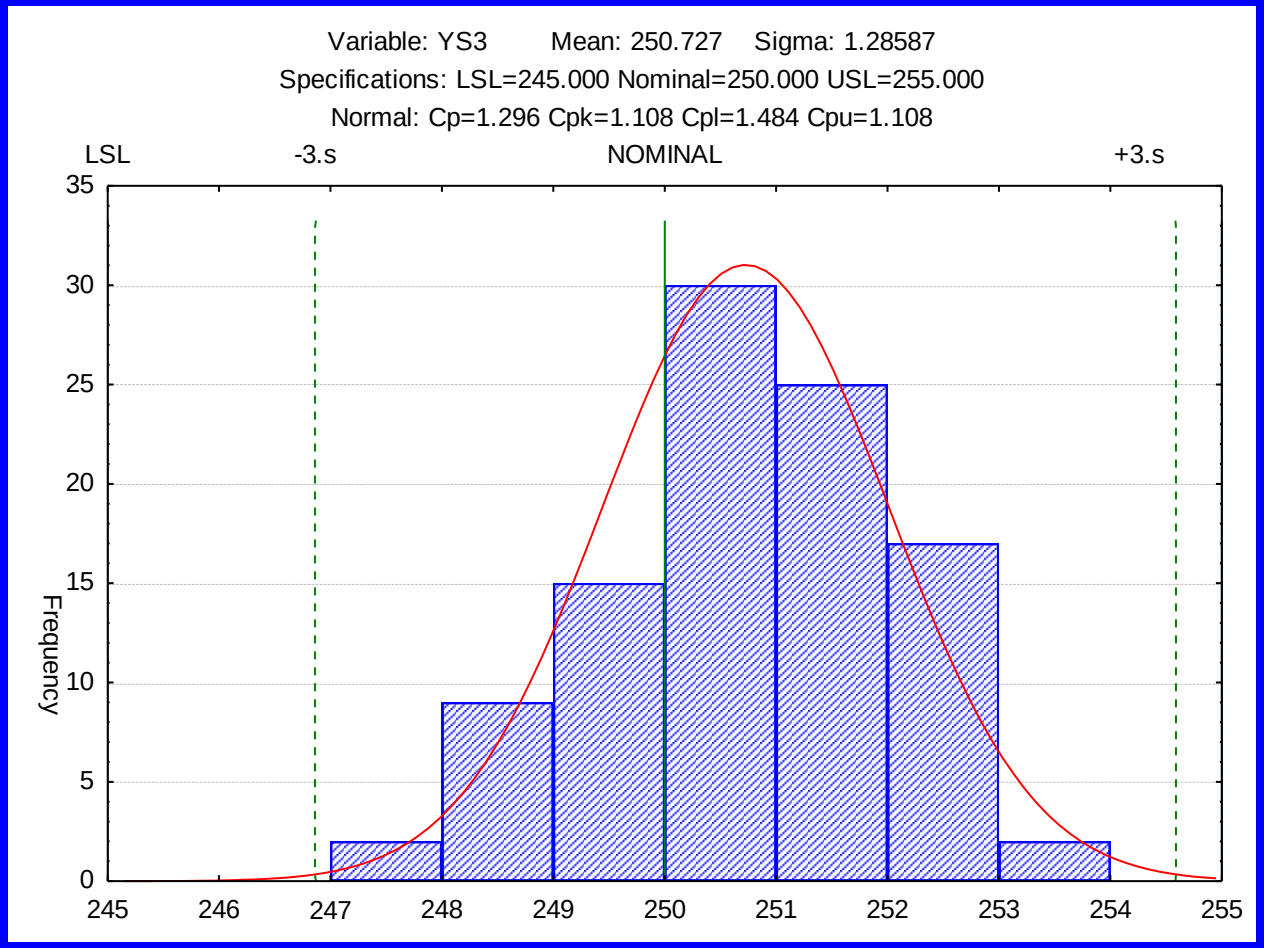
Open cpdata1.sta

Statistics>Industrial Statistics>Process Analysis
Process capability analysis

Variables: YS3
Process specs…



Summary histogram



$$s = \sqrt{\frac{\sum_{i}^{100} (x_i - \bar{x})^2}{99}}$$

μ= 250.727
σ= 1.2859
specs: 250±5

Advanced, normal tab

Descriptive statistics

Descriptive Statistics (cpdata1)
Variable:YS3
N = 100

Statistic	Value
Mean	250.7265010
Median	250.6447161
25th Percentile (Q25)	249.9863486
75th Percentile (Q75)	251.6695272
Minimum Value	247.0155871
Maximum Value	253.6742742
Standard Deviation	1.2858656
Variance	1.6534505
Skewness	-0.3316317
Kurtosis	-0.1017553
Number of samples	20.0000000
Sample size	5.0000000
Sigma-S (R-bar/d2)	1.1167723

Number beyond specs

Variable: YS3, Distribution: Normal (CPDATA1.STA)
Specifications: Lower=245.000 Nominal=250.000 Upper=255.000
Mean:250.73,Std.Dv:1.2859

	Observed	Percent Observed	Expected	Percent Expected
Above USL:	0	0.00	0.044457	0.044457
Below LSL:	0	0.00	0.000423	0.000423
Total	0	0.00	0.044880	0.044880

Example 13

Compare two processes, the specification for both is 100±1.

I. σ=0.2, μ=99.5, that is the center of fluctuation deviates from the nominal value

II. σ=0.4, μ=100, that is the center of fluctuation is the nominal value, but the fluctuation is larger

MSE for the two processes:

Capability indices for the two processes:

C_P=

C_{PU}=

C_{PL}=

C_{PK}=

C_{Pm}=

Example 14

The specification is 100±1, the estimate of σ is s=0.2. Calculate the capability indices and the proportion beyond specs (above *USL* or below *LSL)*, if the estimate of μ is 100, 99.5 and 100.5!

μ	C_P	C_{PU}	C_{PL}	C_{PK}
100.0				
99.5				
100.5				

μ	$z_{\text{fölső}} = \frac{USL - \mu}{\sigma}$	$>USL$	$z_{\text{alsó}} = \frac{LSL - \mu}{\sigma}$	$<LSL$
100.0				
99.5				
100.5				

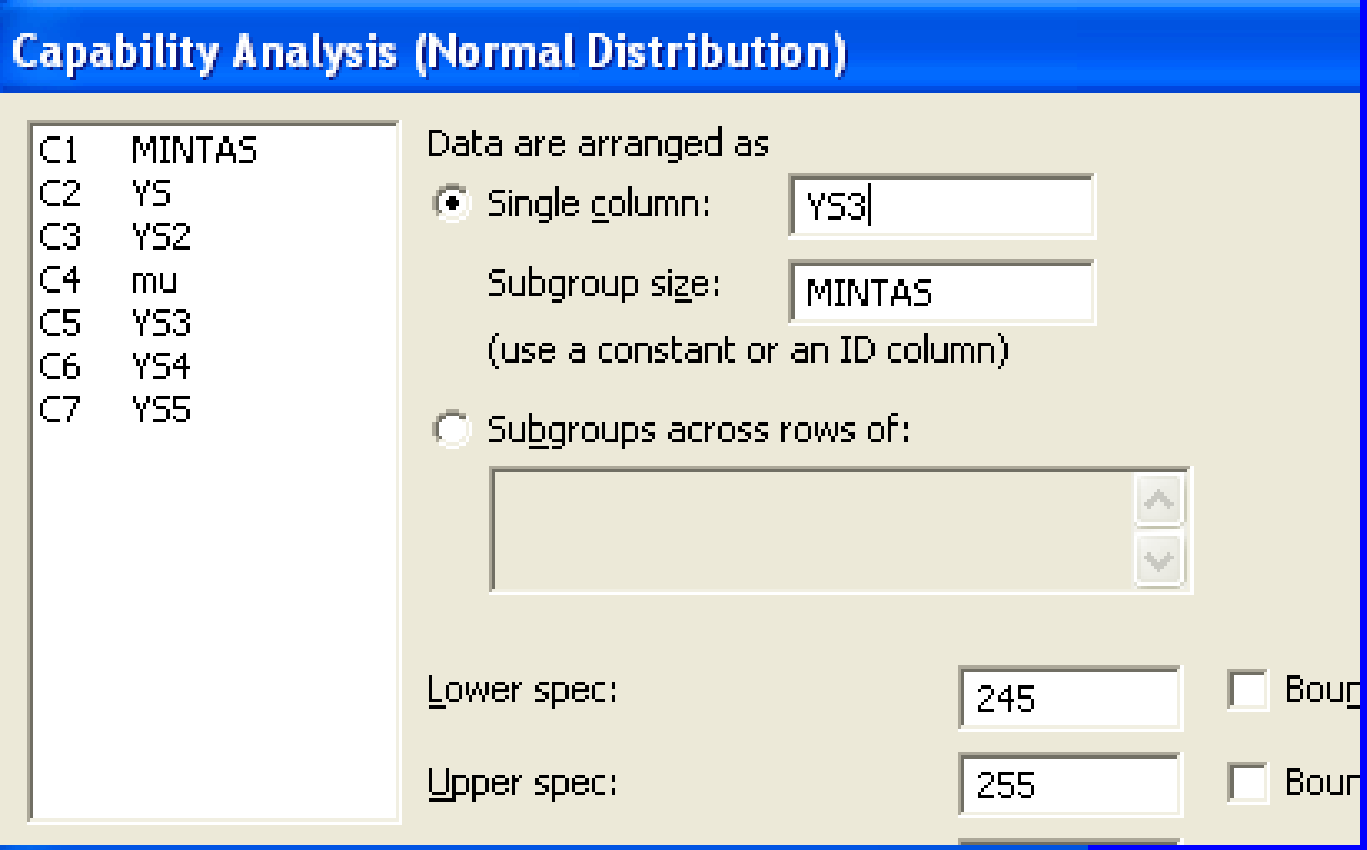
26. példa

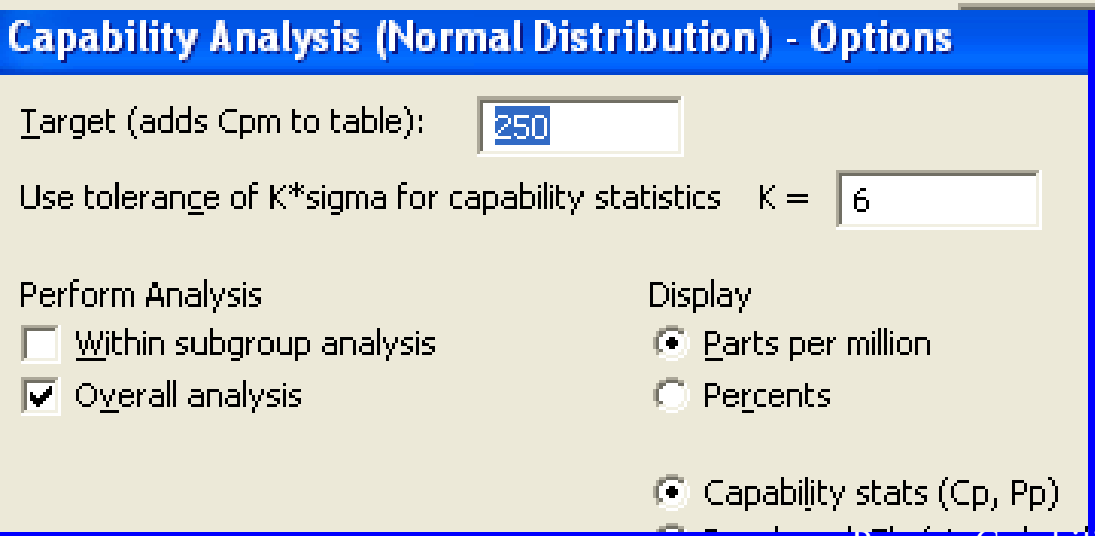
Egy gyártási folyamatban ellenőrző kártya felvételekor kapott adatok a cpdata1.mtw file YS3 oszlopában vannak. Az előírás 250±5 egység.
Számítsuk ki a folyamatképességi indexeket! A termék mekkora része lesz kívül a tűrési tartományon (lesz *USL* fölött ill. *LSL* alatt)?

Minitab>Stat>Quality Tools>Capability Analysis>Normal
Options:

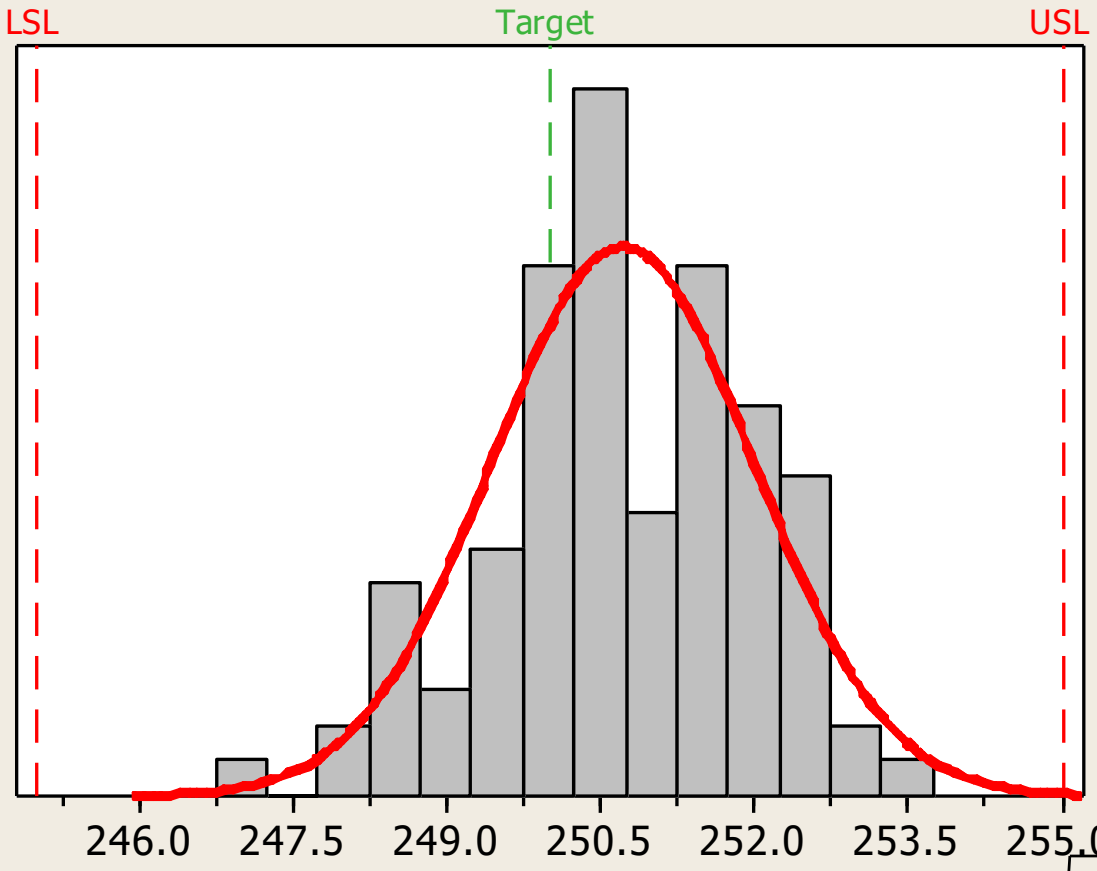
Perform Analysis: Overall analysis

Target


Capability Analysis (Normal Distribution)
C1 MINTAS
C2 YS
C3 YS2
C4 mu
C5 YS3
C6 YS4
C7 YS5
Data are arranged as
Single column: YS3
Subgroup size: MINTAS
(use a constant or an ID column)
Subgroups across rows of:
Lower spec: 245
Upper spec: 255


Capability Analysis (Normal Distribution) - Options
Target (adds Cpm to table): 250
Use tolerance of K*sigma for capability statistics K = 6
Perform Analysis
Within subgroup analysis
Overall analysis
Display
Parts per million
Percents
Capability stats (Cp, Pp)

Process Capability of YS3



Process Data	
LSL	245
Target	250
USL	255
Sample Mean	250.727
Sample N	100
StDev(Overall)	1.28587

Overall Capability	
Pp	1.30
PPL	1.48
PPU	1.11
Ppk	1.11
Cpm	1.13

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Overall Performance	
PPM < LSL	4.23
PPM > USL	444.57
PPM Total	448.79

$$s_{\text{Overall}} = \sqrt{\frac{\sum_{i}^{100}\left(x_i - \bar{x}\right)^2}{99}}$$

Case of one-sided specification

Example 6

A. R. Tenner, I. J. DeToro: Total Quality Management, Addison-Wesley, 1992

Capability analysis for sick leave security payments

Specification: pay out within 14 days

Sicksec.sta

$$C_P = \frac{USL - LSL}{6\sigma}$$

$$C_{PU} = \frac{USL - \mu}{3\sigma}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma}; \quad C_{PK} = \min\left(C_{PU}, C_{PL}\right)$$

Raw data | Grouping | Distribution

Variables: days

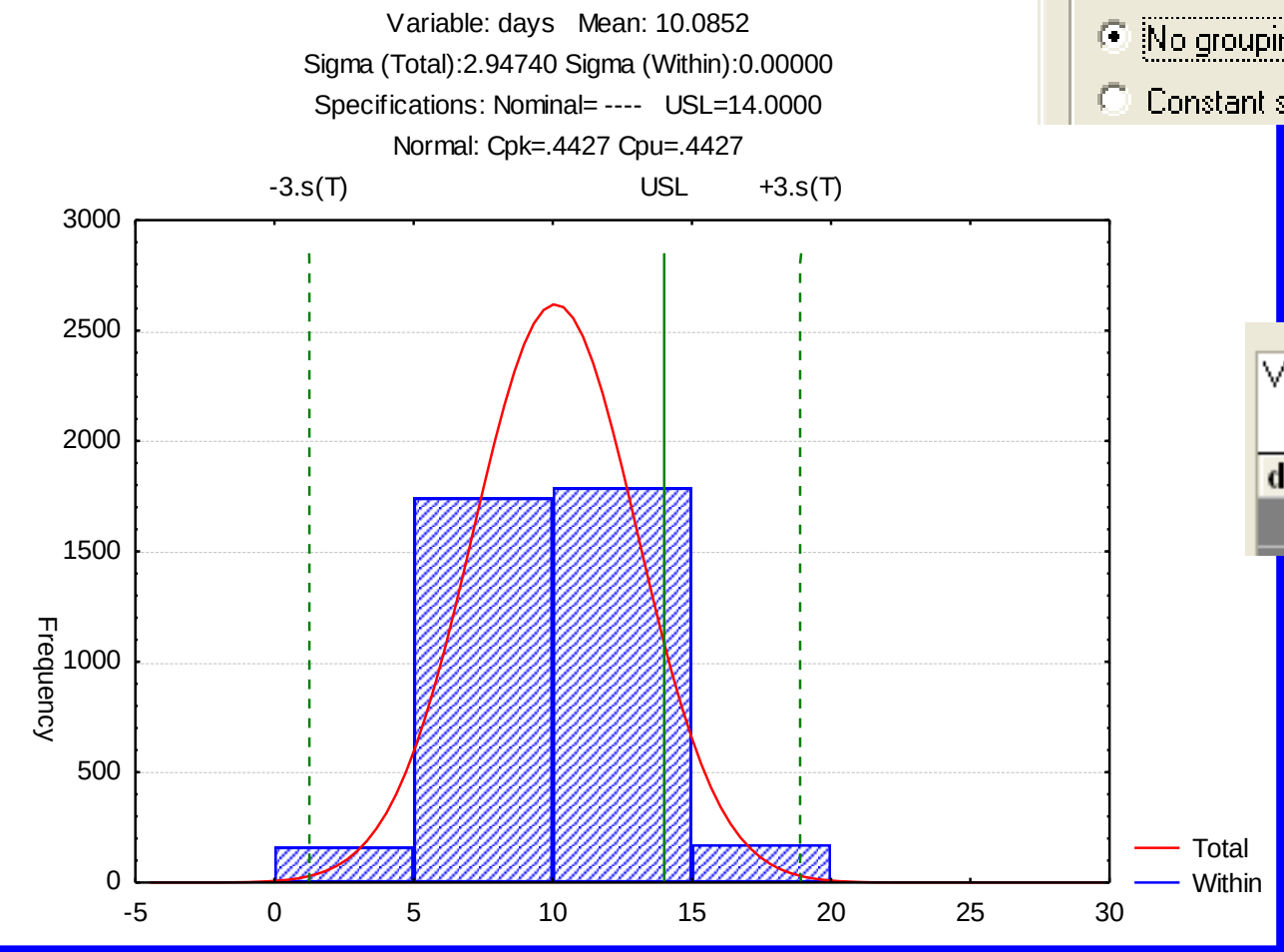
Process specs: Nominal= ---- Upper=14.0000

Grouping (process capability/performance)

(•) No grouping (single sample)

( ) Constant sample size: 1

Select group
specify gro
Process Ca
Performanc



Variable	Nominal	Delta	LSL	USL
days				14

Case of one-sided specification

Example 24

A. R. Tenner, I. J. DeToro: Total Quality Management, Addison-Wesley, 1992

Capability analysis for sick leave security payments

Specification: pay out within 14 days

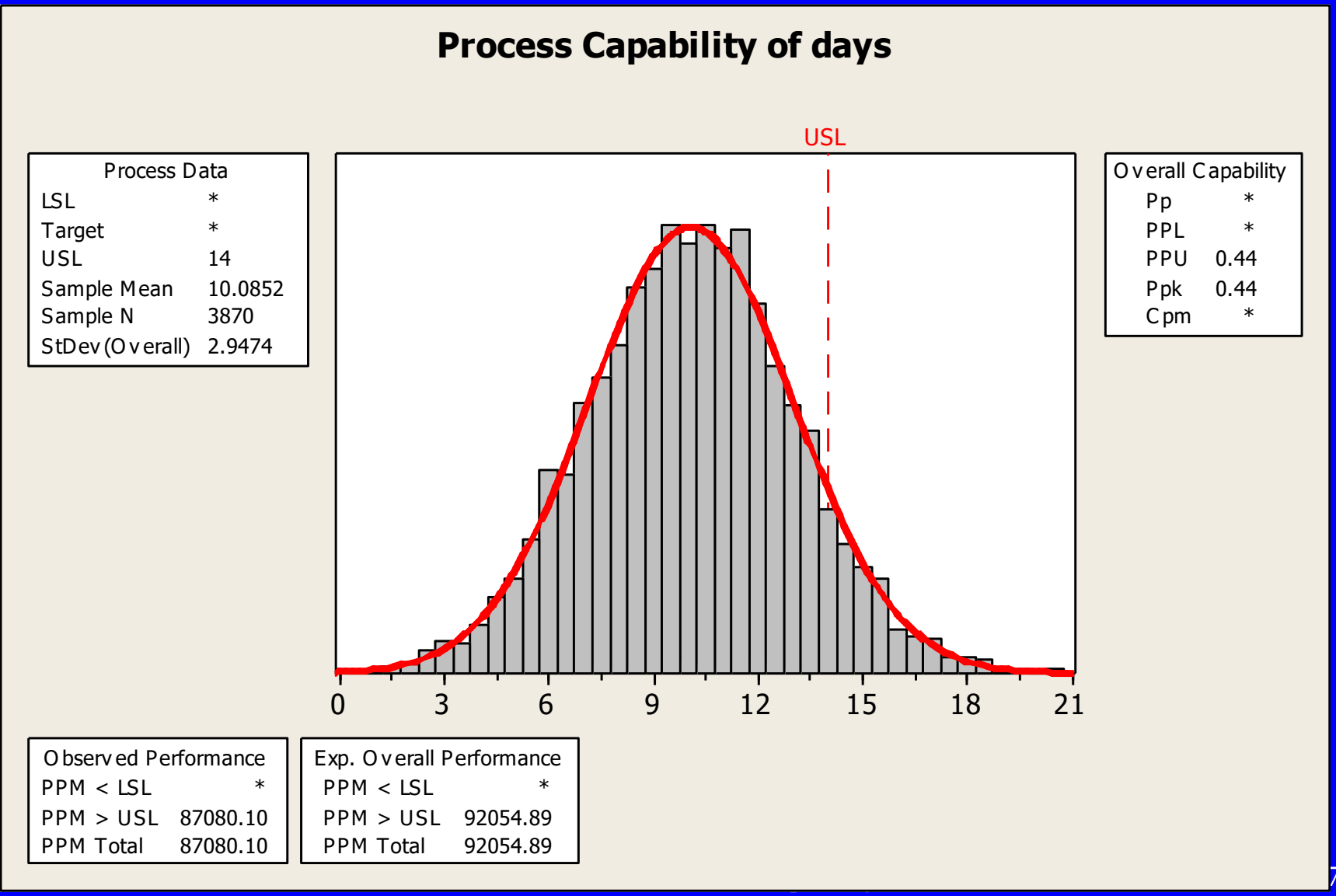
Sicksec.mtw

$$C_P = \frac{USL - LSL}{6\sigma}$$

$$C_{PU} = \frac{USL - \mu}{3\sigma}; \quad C_{PL} = \frac{\mu - LSL}{3\sigma}; \qquad C_{PK} = \min\left(C_{PU}, C_{PL}\right)$$

Minitab>Stat>Quality Tools>Capability Analysis>Normal

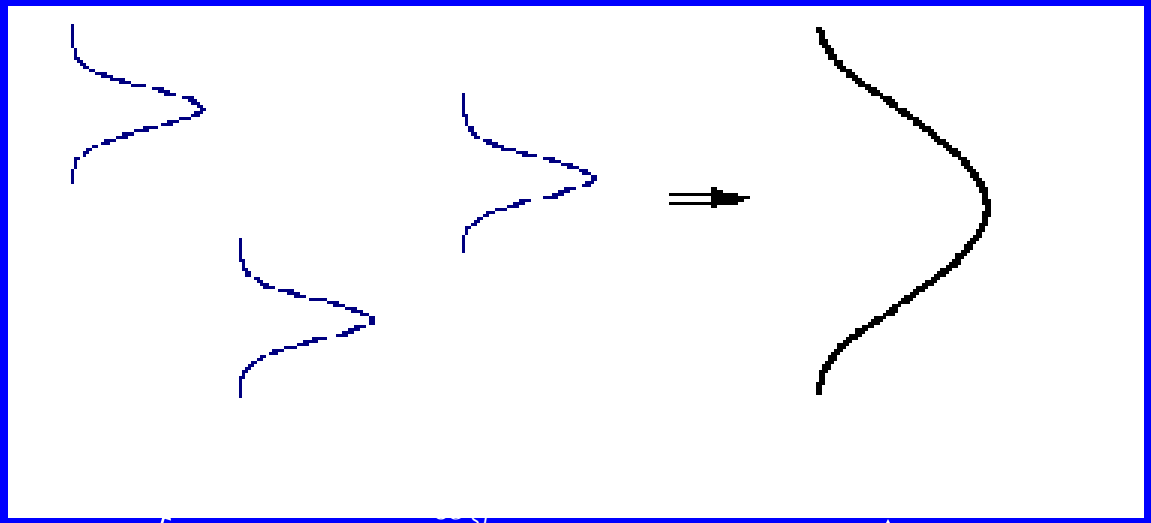
Options: Perform Analysis: Overall analysis



Observed≠Expected

enumerative↔inferential

Process capability and process performance (short term and long term)



$$C_P = \frac{USL - LSL}{6\sigma}$$

which σ ?

σ_{ST}
(short term)

σ_{LT}
(long term)

$$C_P = \frac{USL - LSL}{6\sigma_{ST}}$$

$$P_P = \frac{USL - LSL}{6\sigma_{LT}}$$

Estimating variance from the within-samples (short term) changes refers the internal, random fluctuation $\rightarrow C_P$ (potential capability)

Combining both within-samples and between-samples changes the long term fluctuation is considered $\rightarrow P_P$ (process performance)

$$P_P \leq C_P$$

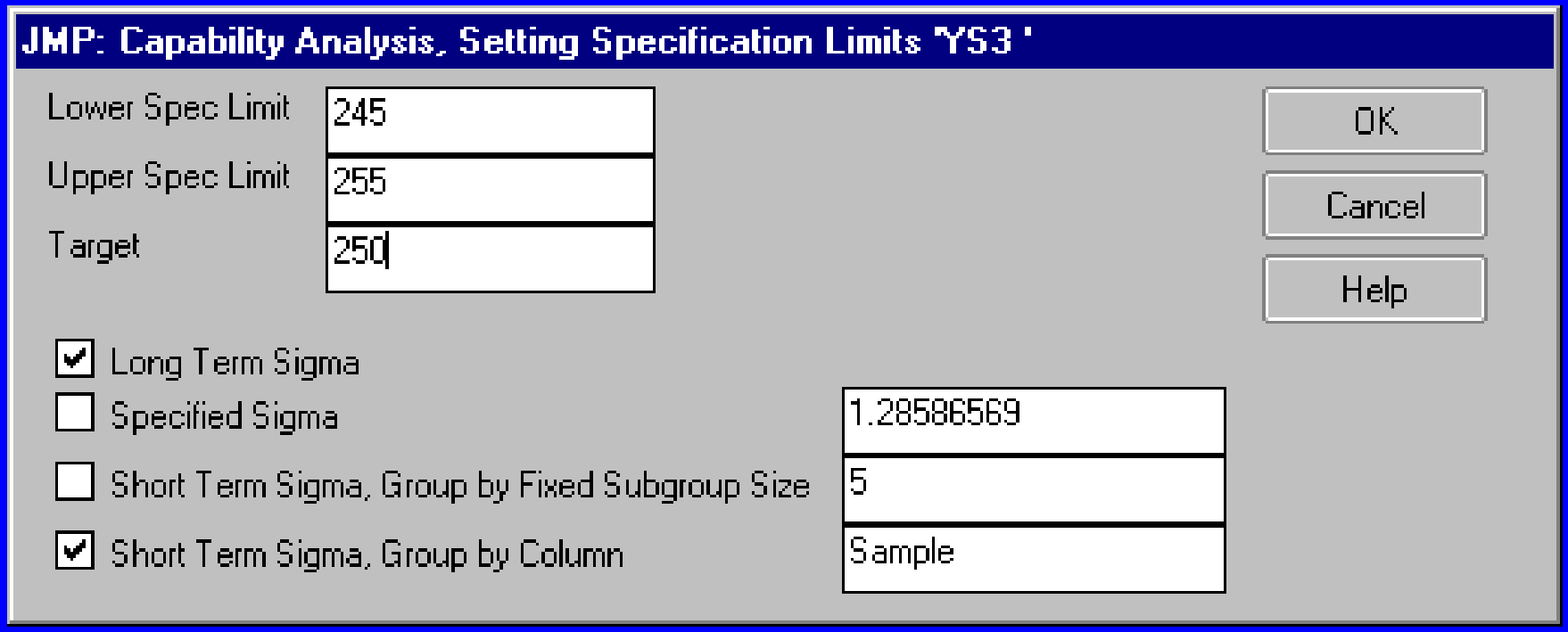
Open Data Table

Chartdata1.xls

Analyze>Distribution

Y, Column: Y3

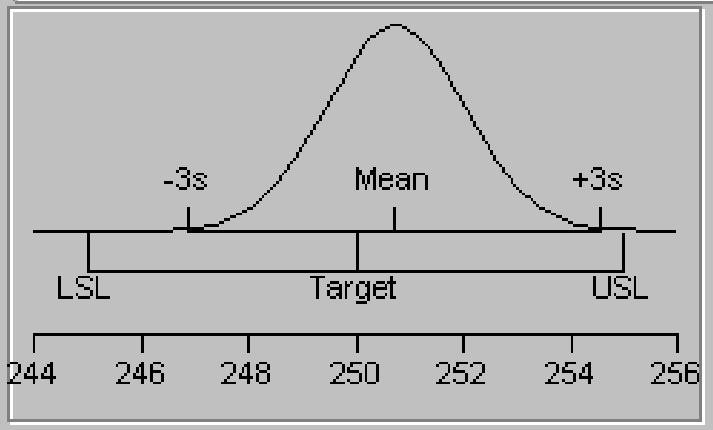
Capability Analysis



YS3

Capability Analysis

Overall, Sigma = 1.28587



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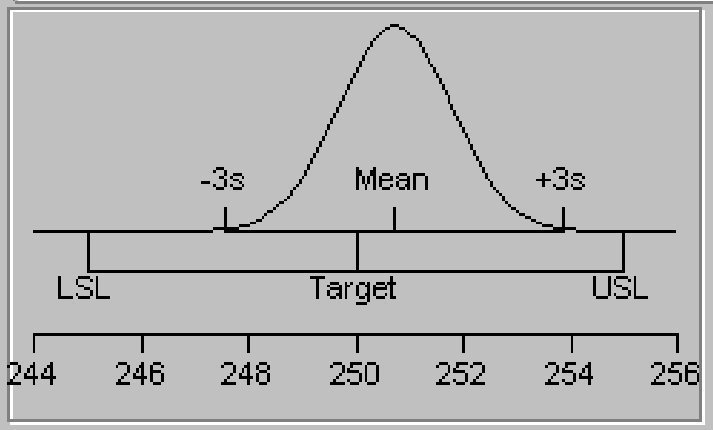

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CPK	1.357	1.157	1.557
CPM	1.306	1.144	1.467
CPL	1.819	1.557	2.079
CPU	1.357	1.157	1.556

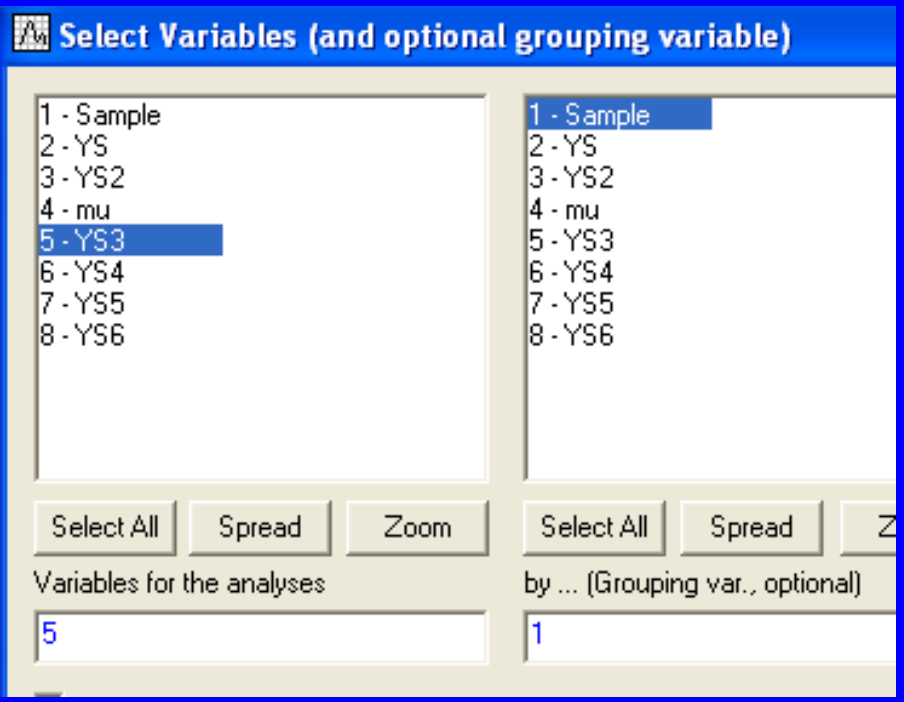
Portion	Percent	PPM
Below LSL	0.0000	0.0244
Above USL	0.0023	23.3566
Total Outside	0.0023	23.3810

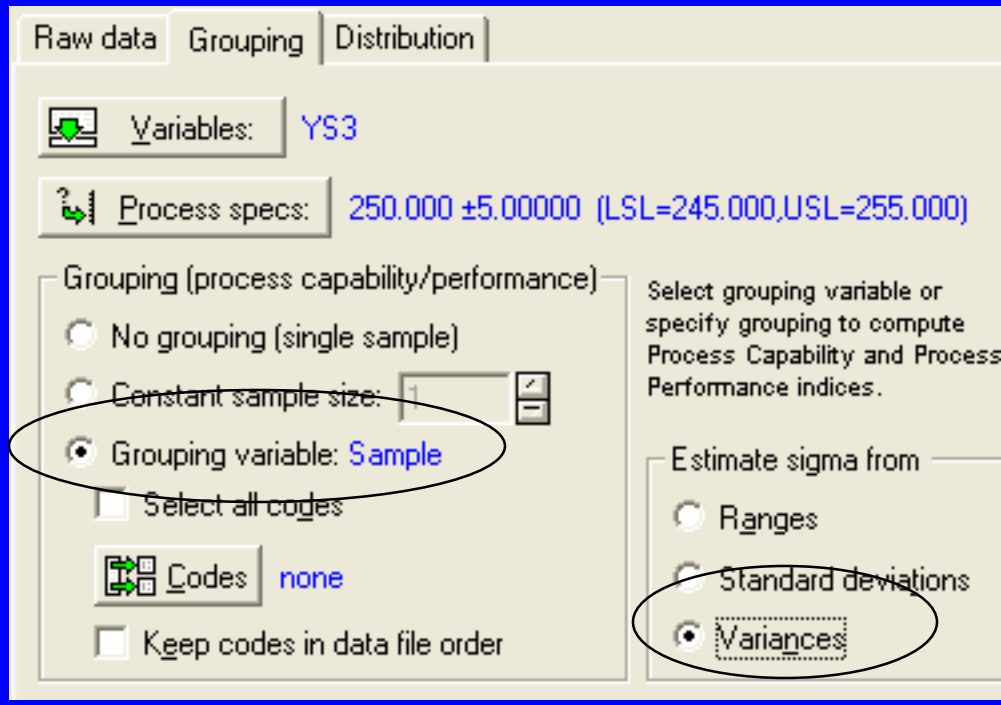
Example 6

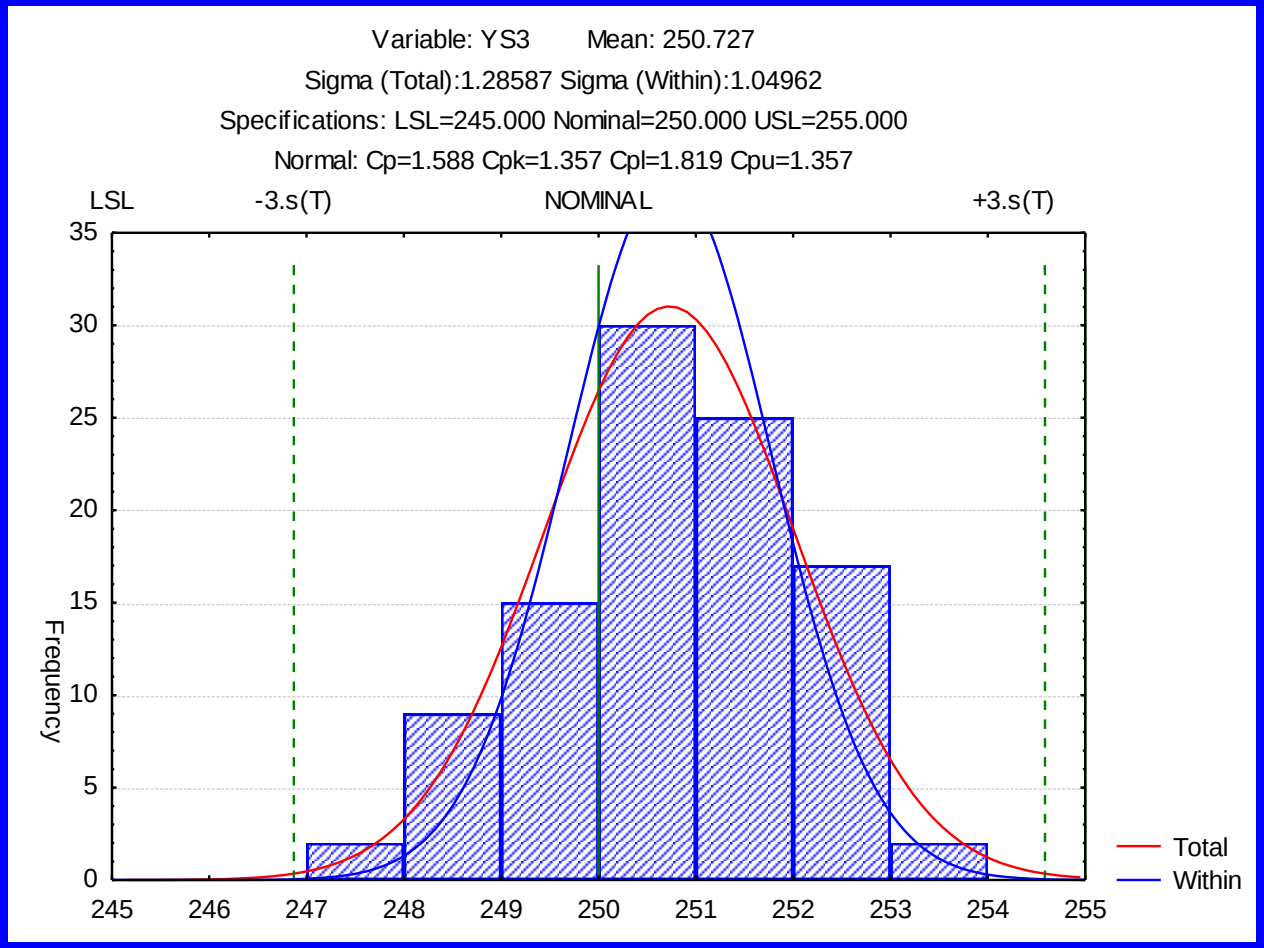
Calculate the performance indices for YS3 data in the dataset cpdata1.sta

Statistics>Industrial Statistics>Process Analysis

Process capability analysis






Variable: YS3 Mean: 250.727
Sigma (Total):1.28587 Sigma (Within):1.04962
Specifications: LSL=245.000 Nominal=250.000 USL=255.000
Normal: Cp=1.588 Cpk=1.357 Cpl=1.819 Cpu=1.357
LSL
-3.s(T)
NOMINAL
+3.s(T)
Frequency
0
5
10
15
20
25
30
35
245
246
247
248
249
250
251
252
253
254
255
Total
Within

Summary: Current variable

Capability Index	Process Capability Z-bench Statistics, Confidence Bounds etc (CPDATA1.STA) Variable: YS3 Value
Cp - Lower CI	1.34213
Cp - Upper CI	1.83320
Cpk - Lower CI	1.13696
Cpk - Upper CI	1.57737
Z - benchmark Potential	4.07125
Z benchmark - LSL	5.45581
Z benchmark - USL	4.07149
Z benchmark - Lower CI	1.79578
Z benchmark - Upper CI	
Overall Process Performance	
PPM < LSL	0.02438
PPM > USL	23.35658
PPM Total	23.38095
Observed Process Performance	
PPM < LSL	0.00000
PPM > USL	0.00000
PPM Total	0.00000
Cpm - Lower CI	0.97565
Cpm - Upper CI	1.27832

Capability Index	Variable: YS3 (CPDATA1.STA) -3.000 *Sigma=247.578 +3.000 *Sigma=253.875 Value
Lower Specification Limit	245.000
Nominal Specification	250.000
Upper Specification Limit	255.000
CP (potential capability)	1.5879
CR (capability ratio)	0.6298
CPK (demonstrated excellence)	1.3572
CPL (lower capability index)	1.8186
CPU (upper capability index)	1.3572
K (non-centering correction)	0.1453
CPM (potential capability II)	1.1271

	Process Capability Z-bench Statistics, Confidence Bounds etc (CPDATA1.STA) Variable: YS3
	Value
Pp - Lower CI	1.115754E+00
Pp - Upper CI	1.476234E+00
Ppk - Lower CI	9.402471E-01
Ppk - Upper CI	1.275380E+00
Z - benchmark Overall	3.320802E+00
Z benchmark - LSL	4.453421E+00
Z benchmark - USL	3.323441E+00
Z benchmark - Lower CI	1.785034E+00
Z benchmark - Upper CI	-3.000000E+30
Potential Process Performance	
PPM < LSL	4.225645E+00
PPM > USL	4.445709E+02
PPM Total	4.487965E+02

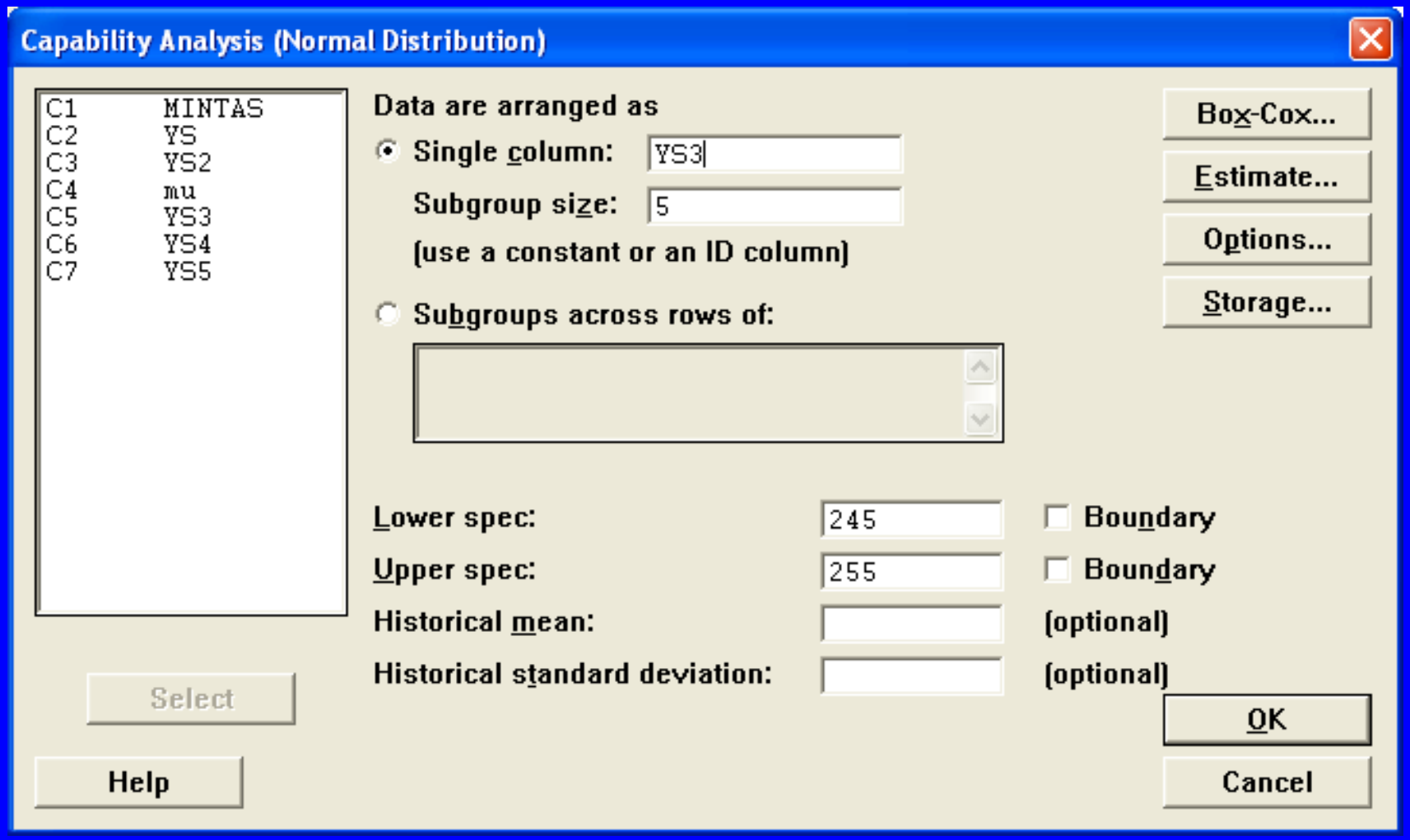
	Variable: YS3 (CPDATA1.STA) -3.000 *Sigma=246.869 +3.000 *Sigma=254.584
	Value
Lower Specification Limit	245.0000
Nominal Specification	250.0000
Upper Specification Limit	255.0000
PP (performance index)	1.2961
PR (performance ratio)	0.7715
PPK (perf. demonstr. excell.)	1.1078
PPL (lower performance index)	1.4845
PPU (upper performance index)	1.1078

5. példa

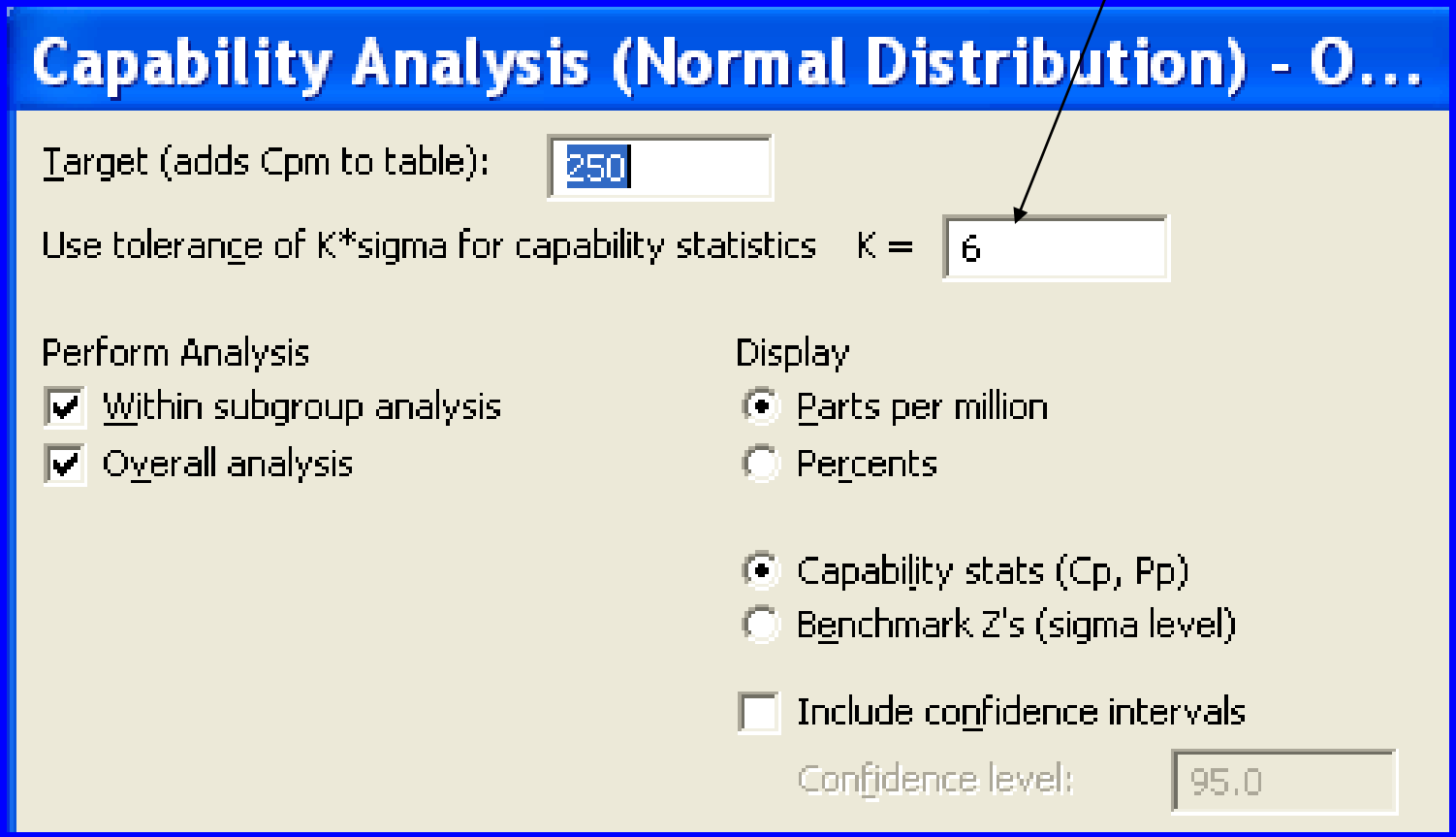
Example 6

Calculate the capability and performance indices together with the expected proportion of non-conforming items for YS3 data in the dataset cpdata1.mtw!
The specification is 250±5 unit.

Minitab>Stat>Quality Tools>Capability Analysis (Normal)>Options:
Perform Analysis: Within subgroup analysis, Overall analysis
Target

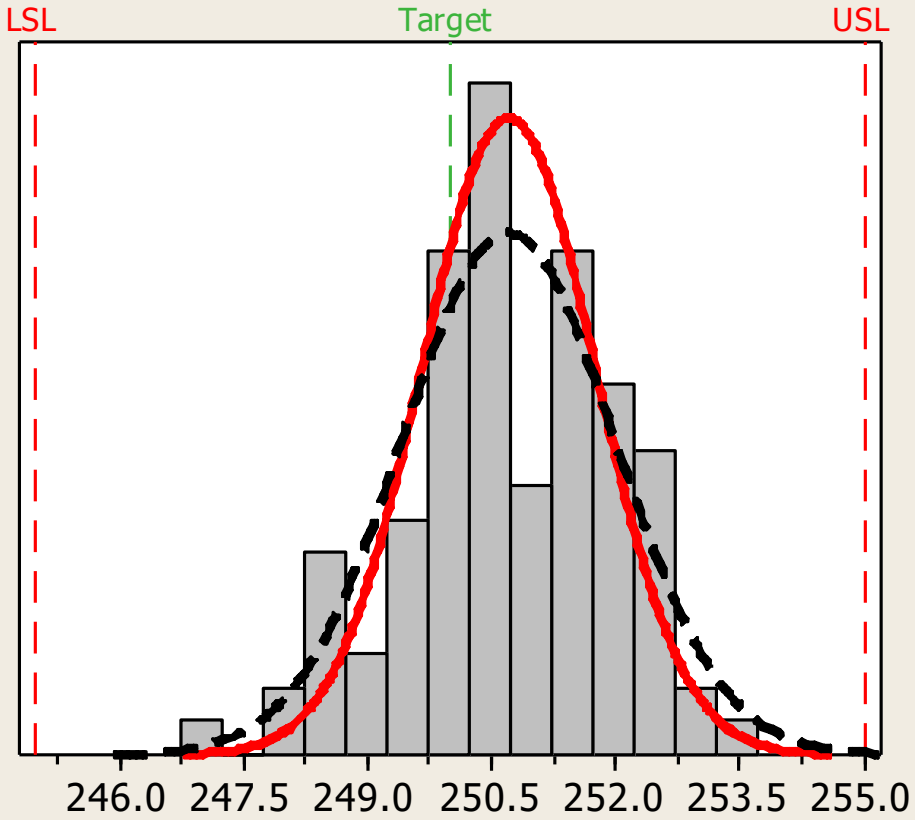

Capability Analysis (Normal Distribution)
C1 MINTAS
C2 YS
C3 YS2
C4 mu
C5 YS3
C6 YS4
C7 YS5
Data are arranged as
Single column: YS3
Subgroup size: 5
(use a constant or an ID column)
Subgroups across rows of:
Box-Cox...
Estimate...
Options...
Storage...
Lower spec: 245
Boundary
Upper spec: 255
Boundary
Historical mean:
(optional)
Historical standard deviation:
(optional)
Select
OK
Help
Cancel

$$C_P = \frac{USL - LSL}{6\sigma}$$



$\mu + \varepsilon_{between} + \varepsilon_{within}$ $\mu = 250.5$ $\varepsilon_{between} \sim N(0,1)$ $\varepsilon_{within} \sim N(0,1)$

Process Capability of YS3



Process Data	
LSL	245
Target	250
USL	255
Sample Mean	250.727
Sample N	100
StDev(Within)	1.0529
StDev(Overall)	1.28587

Within
Overall

Potential (Within) Capability	
Cp	1.58
CPL	1.81
CPU	1.35
Cpk	1.35

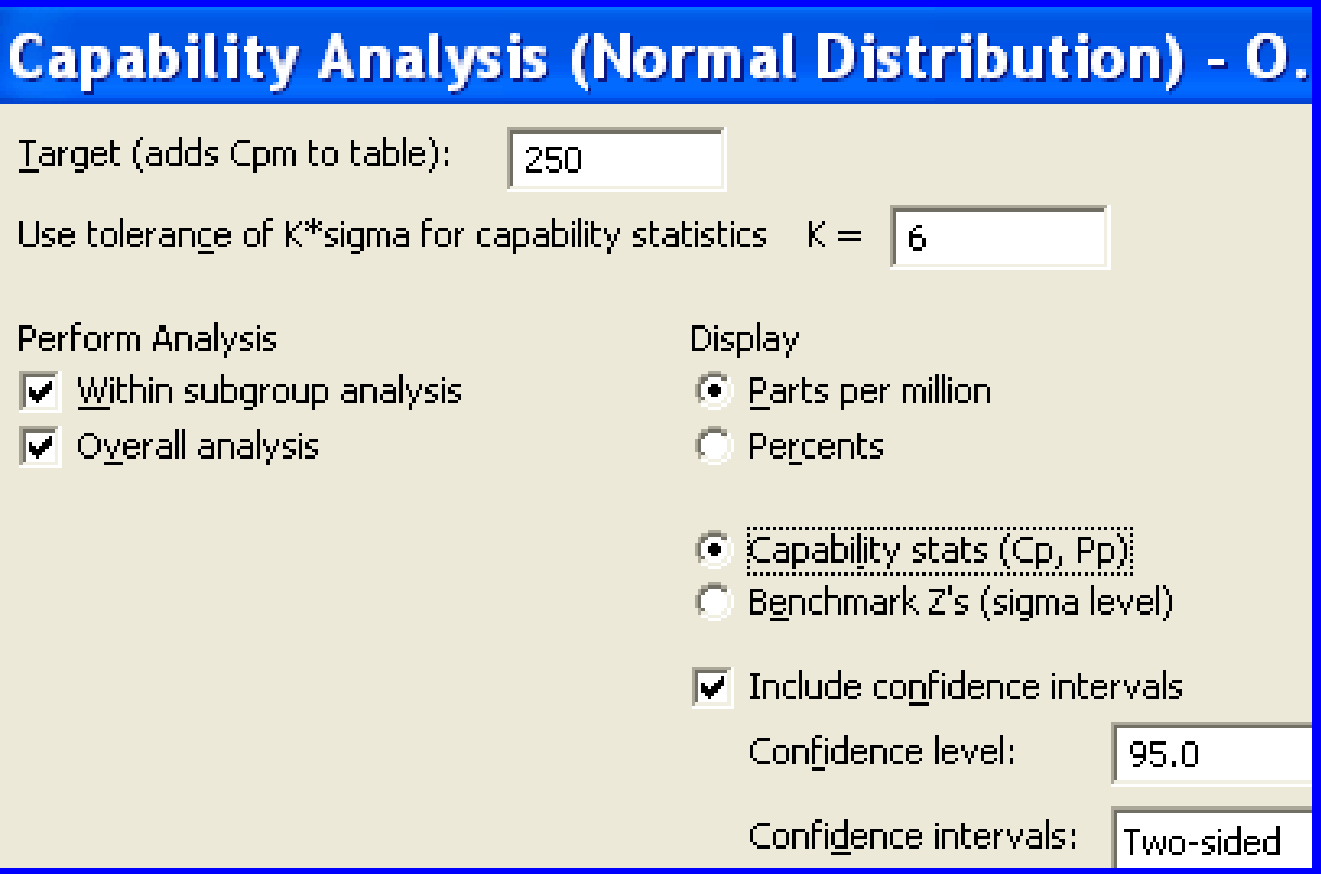
Overall Capability	
Pp	1.30
PPL	1.48
PPU	1.11
Ppk	1.11
Cpm	1.13

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Within Performance	
PPM < LSL	0.03
PPM > USL	24.66
PPM Total	24.69

Exp. Overall Performance	
PPM < LSL	4.23
PPM > USL	444.57
PPM Total	448.79

The indices computed from sample data are random variables, give their confidence interval as well!

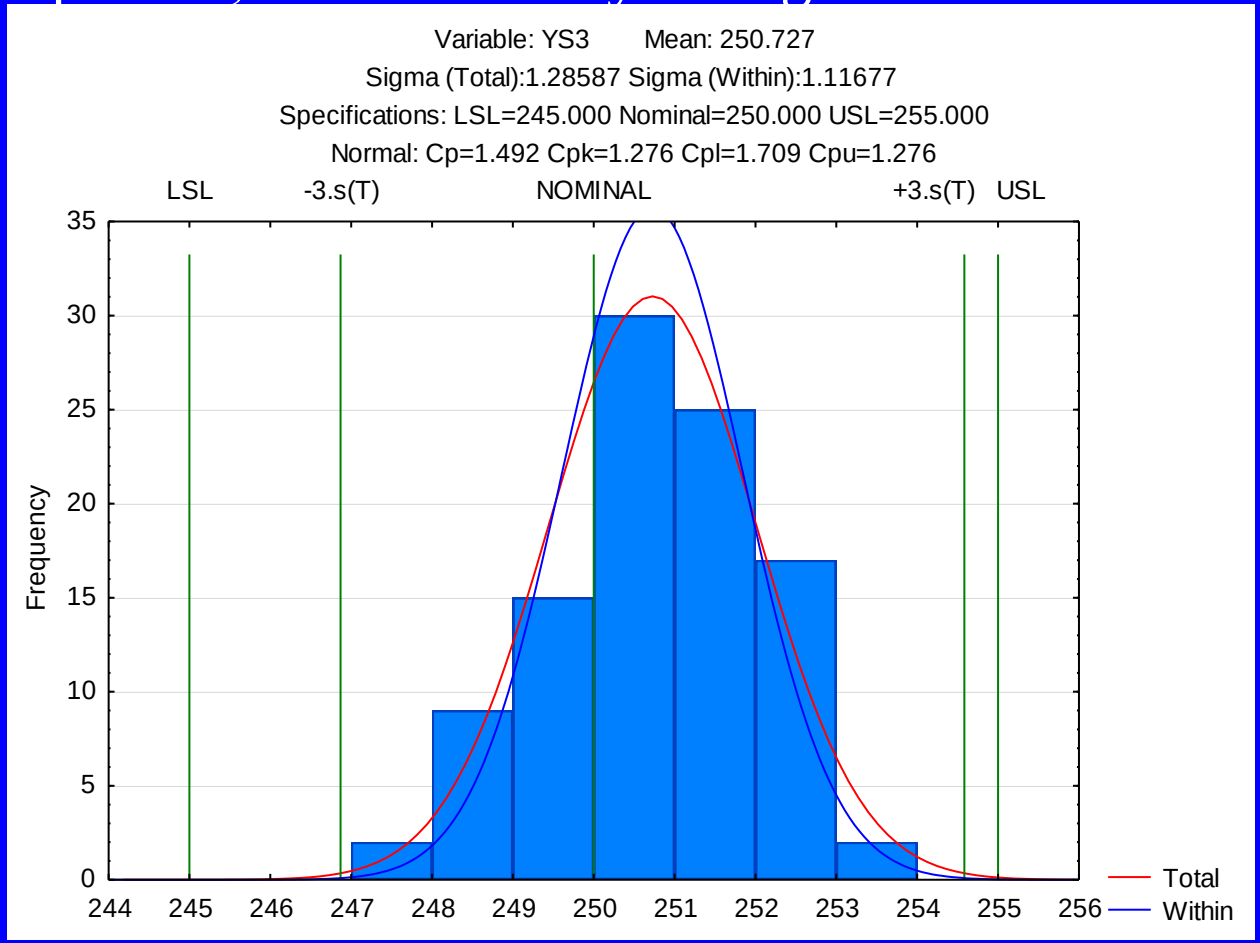


only sampling uncertainty is considered, meaurement uncertainty is disregarded

Open cpdata1.sta Statistics>Industrial Statistics>Process Analysis
Process capability analysis

Variables: YS3, by MINTAS

Process specs…, Summary histogram



Statistics>Industrial Statistics>Process Analysis

Process capability analysis

Summary: Current variable

Capability Index Within-sample sigma=R-bar/d2	YS3; Set 0 (Default Set) (cpdata1) -3.000 *Sigma= 247.37618 3.000 *Sigma= 254.07682
	Value
Lower Specification Limit	245.0000
Nominal Specification	250.0000
Upper Specification Limit	255.0000
CP (potential capability)	1.4924
CR (capability ratio)	0.6701
CPK (demonstrated excellence)	1.2756
CPL (lower capability index)	1.7092
CPU (upper capability index)	1.2756
K (non-centering correction)	0.1453

Performance Index	YS3; Set 0 (Default Set) (cpdata1) -3.000 *Sigma= 246.86890 3.000 *Sigma= 254.58410
	Value
Lower Specification Limit	245.0000
Nominal Specification	250.0000
Upper Specification Limit	255.0000
PP (performance index)	1.2961
PR (performance ratio)	0.7715
PPK (perf. demonstr. excell.)	1.1078
PPL (lower performance index)	1.4845
PPU (upper performance index)	1.1078

Statistics>Industrial Statistics>Process Analysis

Process capability analysis

Variables: YS3, by MINTAS

Process specs…, Advanced, normal fül

Descriptive statistics Number beyond specs

Statistic	Descriptive Statistics (cpdata1) Variable:YS3 N = 100 Value
Mean	250.7265010
Median	250.6447161
25th Percentile (Q25)	249.9863486
75th Percentile (Q75)	251.6695272
Minimum Value	247.0155871
Maximum Value	253.6742742
Standard Deviation	1.2858656
Variance	1.6534505
Skewness	-0.3316317
Kurtosis	-0.1017553
Number of samples	20.0000000
Sample size	5.0000000
Sigma-S (R-bar/d2)	1.1167723

	Variable: YS3, Distribution: Normal (cpdata1) Specifications: Lower=245.000 Nominal=250.000 Upper=255.00 Mean:250.73,Std.Dv:1.2859			
	Observed	Percent Observed	Expected	Percent Expected
Above USL:	0	0.00	0.044457	0.044457
Below LSL:	0	0.00	0.000423	0.000423
Total	0	0.00	0.04488	0.04488

Open cpdata1.sta Statistics>Industrial Statistics>Process Analysis
Process capability analysis

Variables: YS3, by MINTAS

Process specs…, Advanced, normal fül

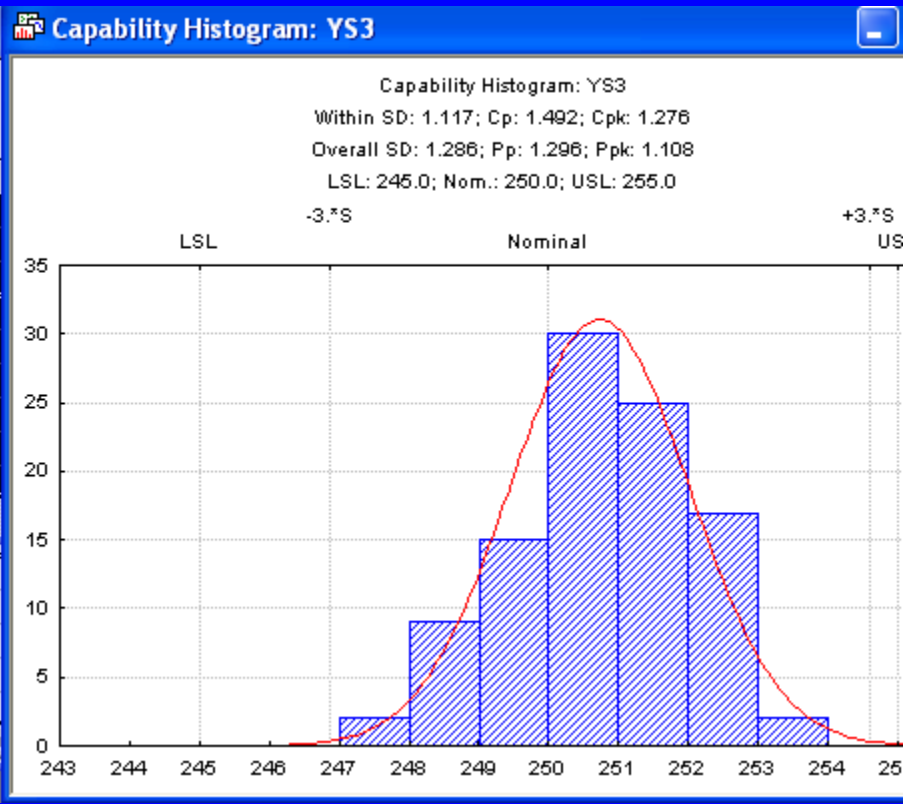
Summary: Current variable, Descriptive statistics,
Summary histogram

Data: Descriptive St...

Statistic	Descriptive Variable: YS N = 100 Value
Mean	250.7265
Median	250.6447
25th Percentile (Q25)	249.9863
75th Percentile (Q75)	251.6695
Minimum Value	247.0156
Maximum Value	253.6743
Standard Deviation	1.2859
Variance	1.6535
Skewness	-0.3316
Kurtosis	-0.1018
Number of samples	20.0000
Sample size	5.0000
Sigma-S (R-bar/d2)	1.1168

Data: YS3; Set 0 (Default Set) (PLD4_1...

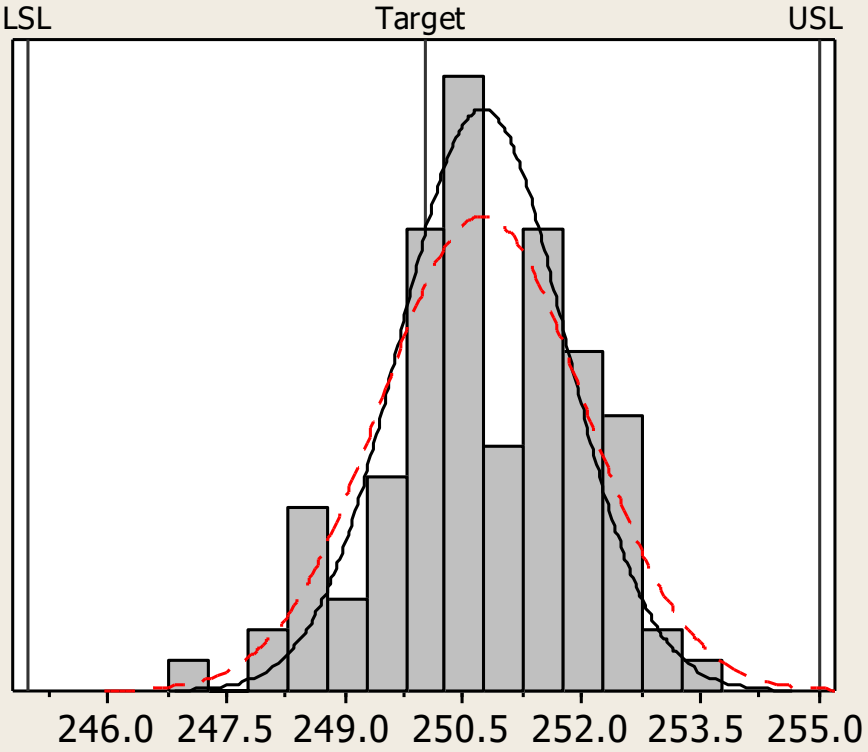
Capability Index Within-sample sigma=R-bar/d2	YS3; Set -3.000 *S 3.000 *Si Value
Lower Specification Limit	245.000
Nominal Specification	250.000
Upper Specification Limit	255.000
CP (potential capability)	1.492
CR (capability ratio)	0.670
CPK (demonstrated excellence)	1.276
CPL (lower capability index)	1.709
CPU (upper capability index)	1.276
K (non-centering correction)	0.145
PP (performance index)	1.296
PR (performance ratio)	0.772
PPK (perf. demonstr. excell.)	1.108
PPL (lower performance index)	1.484
PPU (upper performance index)	1.108



$\mu + \varepsilon_{between} + \varepsilon_{within}$ $\mu = 250.5$ $\varepsilon_{between} \sim N(0,1)$ $\varepsilon_{within} \sim N(0,1)$

Process Capability of YS3

(using 95.0% confidence)



Process Data	
LSL	245.00000
Target	250.00000
USL	255.00000
Sample Mean	250.72650
Sample N	100
StDev(Within)	1.05290
StDev(Overall)	1.28912

Within
Overall

Potential (Within) Capability	
Cp	1.58
Lower CL	1.34
Upper CL	1.83
CPL	1.81
CPU	1.35
Cpk	1.35
Lower CL	1.13
Upper CL	1.57
CCpk	1.58

Overall Capability	
Pp	1.29
Lower CL	1.11
Upper CL	1.47
PPL	1.48
PPU	1.11
Ppk	1.11
Lower CL	0.94
Upper CL	1.27
Cpm	1.13
Lower CL	0.98

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Within Performance	
PPM < LSL	0.03
PPM > USL	24.66
PPM Total	24.69

Exp. Overall Performance	
PPM < LSL	4.45
PPM > USL	458.11
PPM Total	462.57

The indices computed from sample data are random variables, give their confidence interval as well!

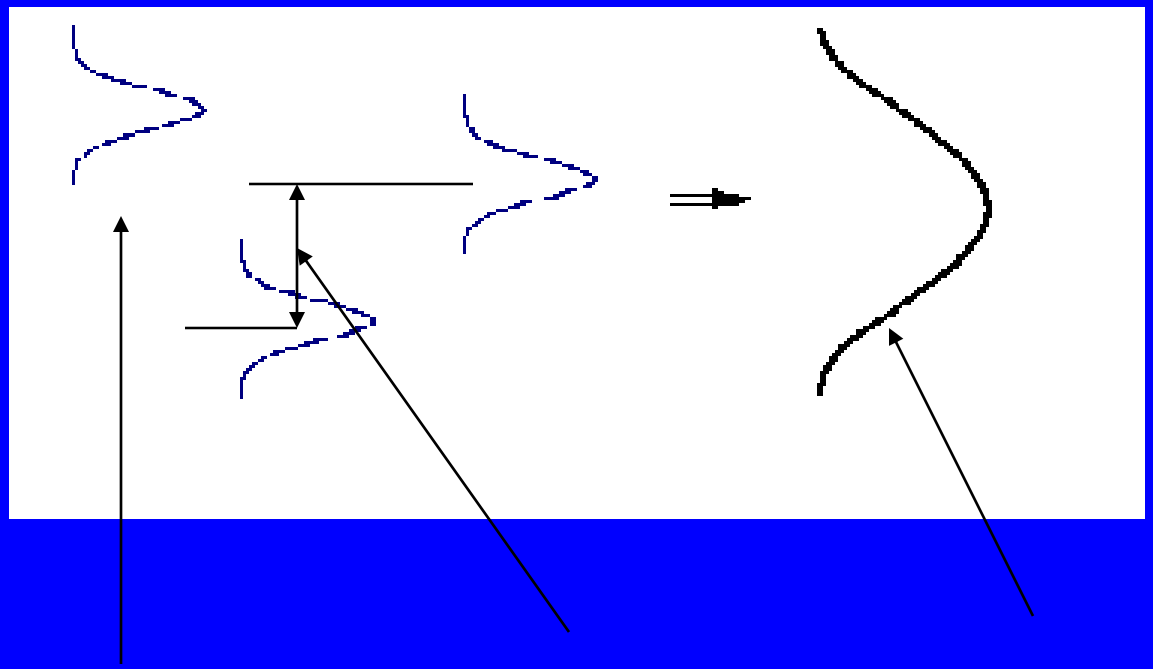
Advanced, normal tab Summary: Current variable

Capability Index	Process Capability (CPDATA1.STA) Variable: YS3 Value
Cp - Lower CI	1.24897
Cp - Upper CI	1.73537
Cpk - Lower CI	1.05721
Cpk - Upper CI	1.49389
Z - benchmark Potential	3.82610
Z benchmark - LSL	5.12772
Z benchmark - USL	3.82665
Z benchmark - Lower CI	1.79471
Z benchmark - Upper CI	
Overall Process Performance	
PPM < LSL	0.14663
PPM > USL	64.94899
PPM Total	65.09563
Observed Process Performance	
PPM < LSL	
PPM > USL	
PPM Total	
Cpm - Lower CI	0.97565
Cpm - Upper CI	1.27832

	Process Capability Variable: YS3 Value
Pp - Lower CI	1.1158
Pp - Upper CI	1.4762
Ppk - Lower CI	0.9402
Ppk - Upper CI	1.2754
Z - benchmark Overall	3.3208
Z benchmark - LSL	4.4534
Z benchmark - USL	3.3234
Z benchmark - Lower CI	1.7850
Z benchmark - Upper CI	
Potential Process Performance	
PPM < LSL	4.2256
PPM > USL	444.5709
PPM Total	448.7965

only sampling uncertainty is considered, meaurement uncertainty is disregarded

$\mu + \varepsilon_{\text{between}} + \varepsilon_{\text{within}}$ $\mu = 250.5$ $\varepsilon_{\text{between}} \sim N(0,1)$ $\varepsilon_{\text{within}} \sim N(0,1)$

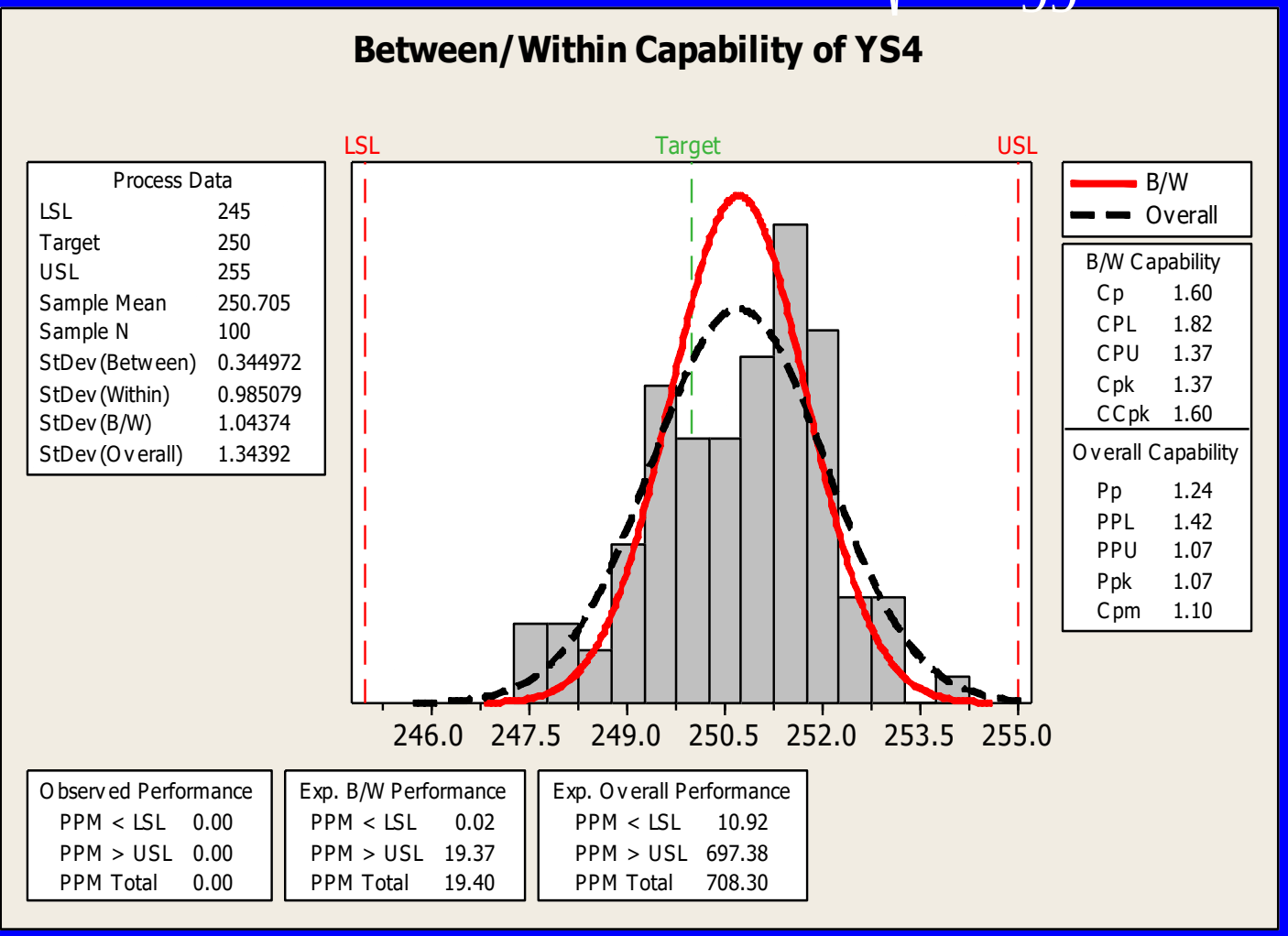


within between overall

Between/Within

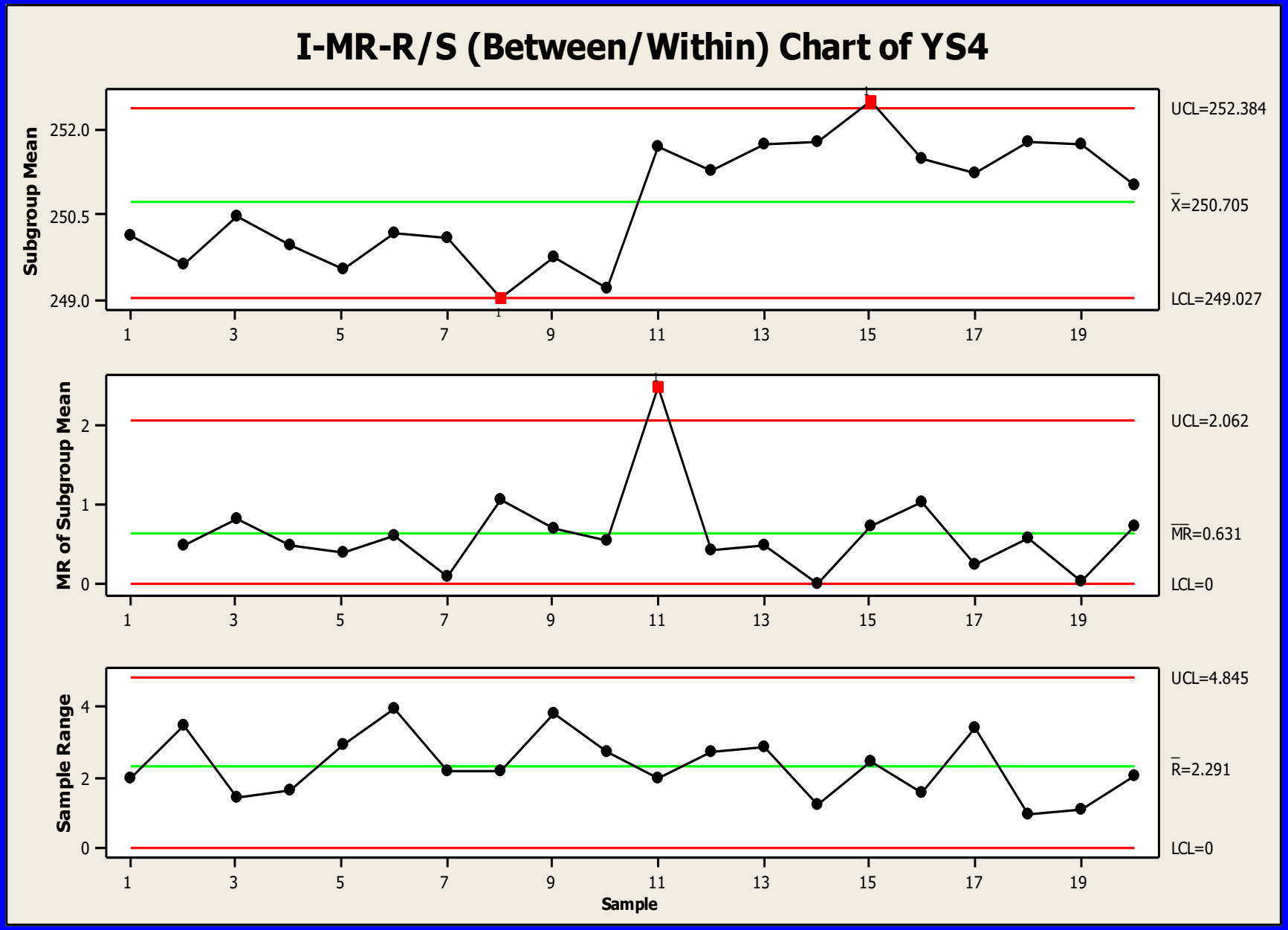
$$s_{\text{Overall}} = \sqrt{\frac{\sum_{i}^{100}(x_i - \bar{x})^2}{99}}$$

$$s_{\text{B/W}} = \sqrt{s_{\text{B}}^2 + s_{\text{W}}^2}$$



overall and B/W are not identical

How to estimate?



The variance components

Machine and process capability

$$C_M = \frac{USL - LSL}{8\sigma} \qquad C_P = \frac{USL - LSL}{6\sigma}$$

The process contains several sub-processes/machines.
To maintain acceptable C_P (e.g. 1.67), the sub-processes should be better.

A toaster consists of 20 parts. If the proportion of nonconforming is 63.4ppm for each, how much is the C_P for a well centered process?

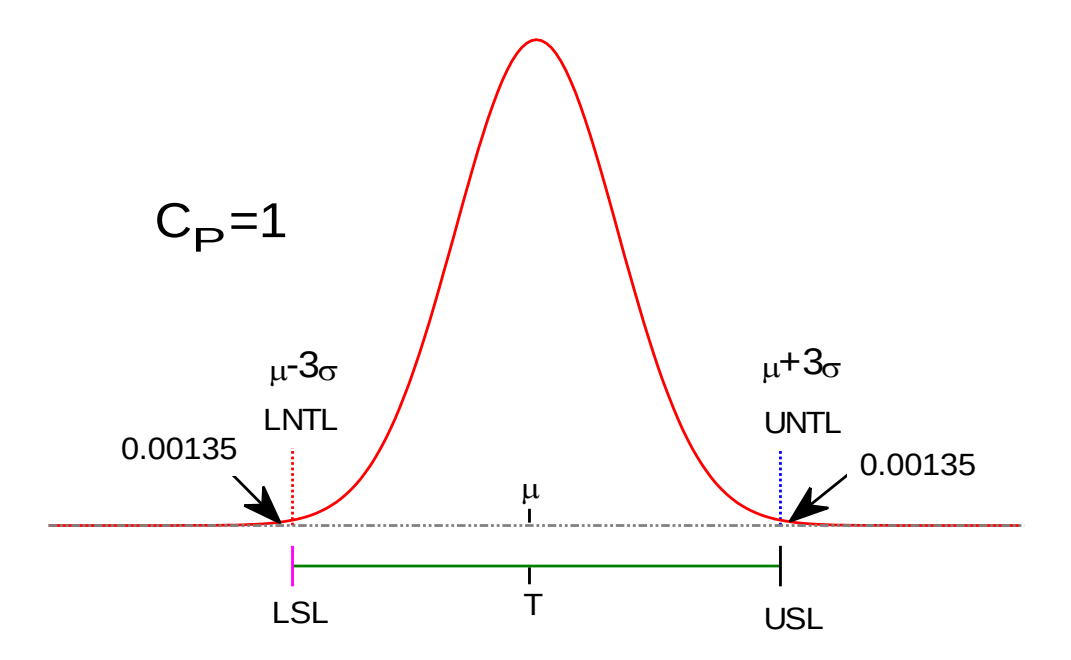
$$C_P = \frac{USL - LSL}{6\sigma} = \frac{z_{USL} - z_{LSL}}{6}$$

$$z = \frac{x - \mu}{\sigma}$$

How much is the proportion of nonconforming for the toaster?

How much is C_p in case of well centered process, based on the proportion of nonconforming?

Non-normal distributions

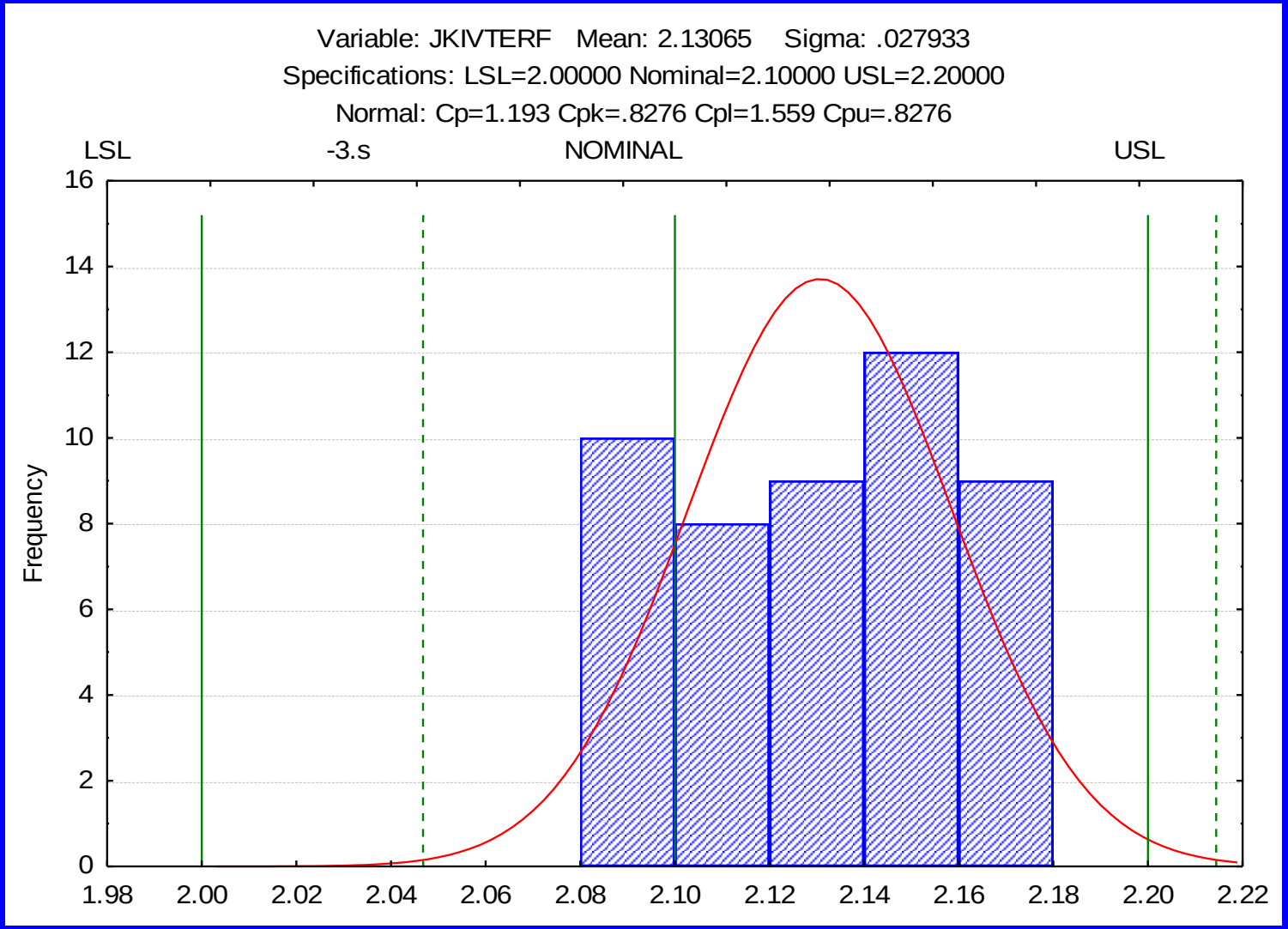


The process capability characterizes the proportion of non-conforming items.

If C_P =1 and the process is centered, the proportion of non-conforming is 0.0027.

Example 12
Ampoule filling process

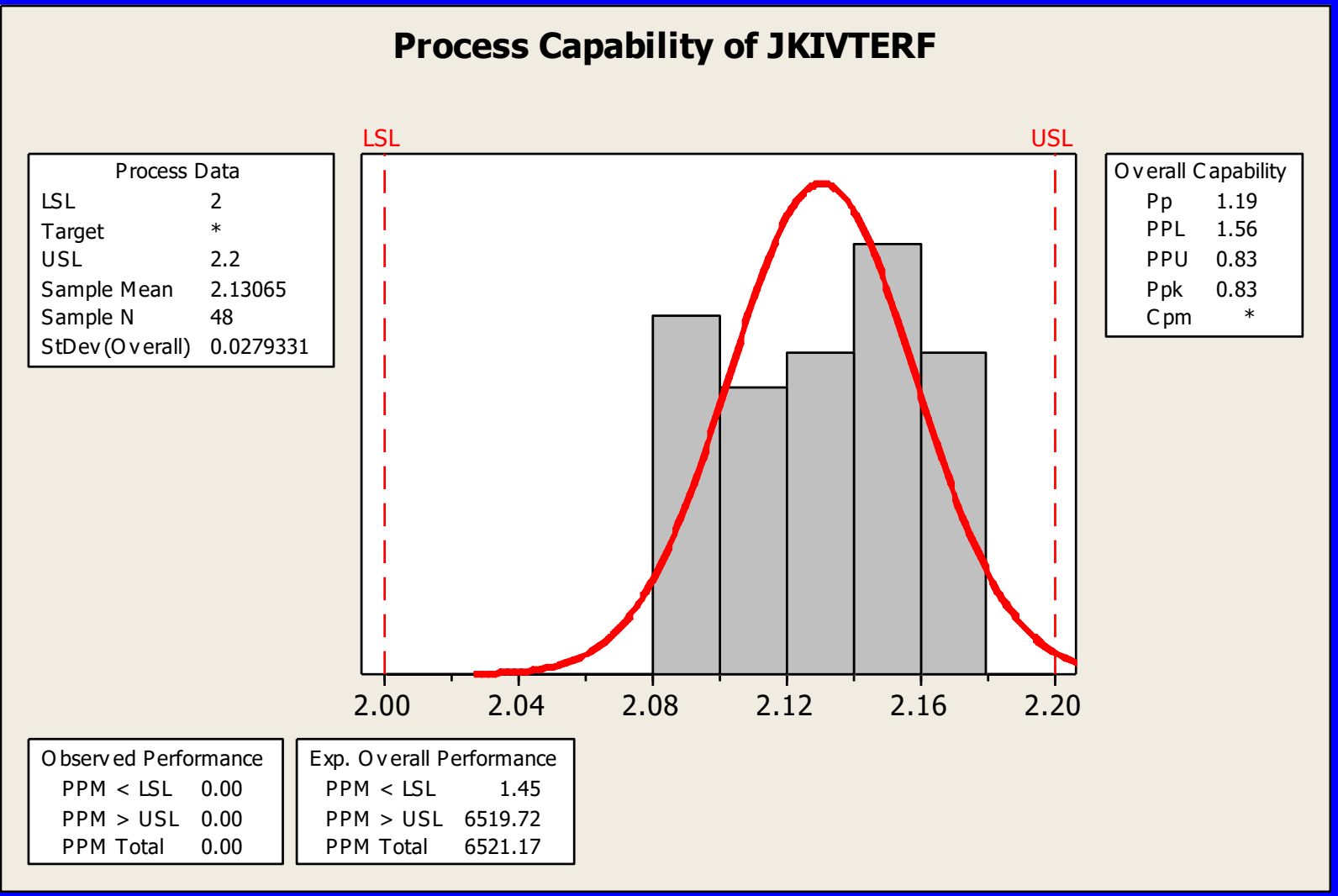
USL=2.2
LSL=2.0

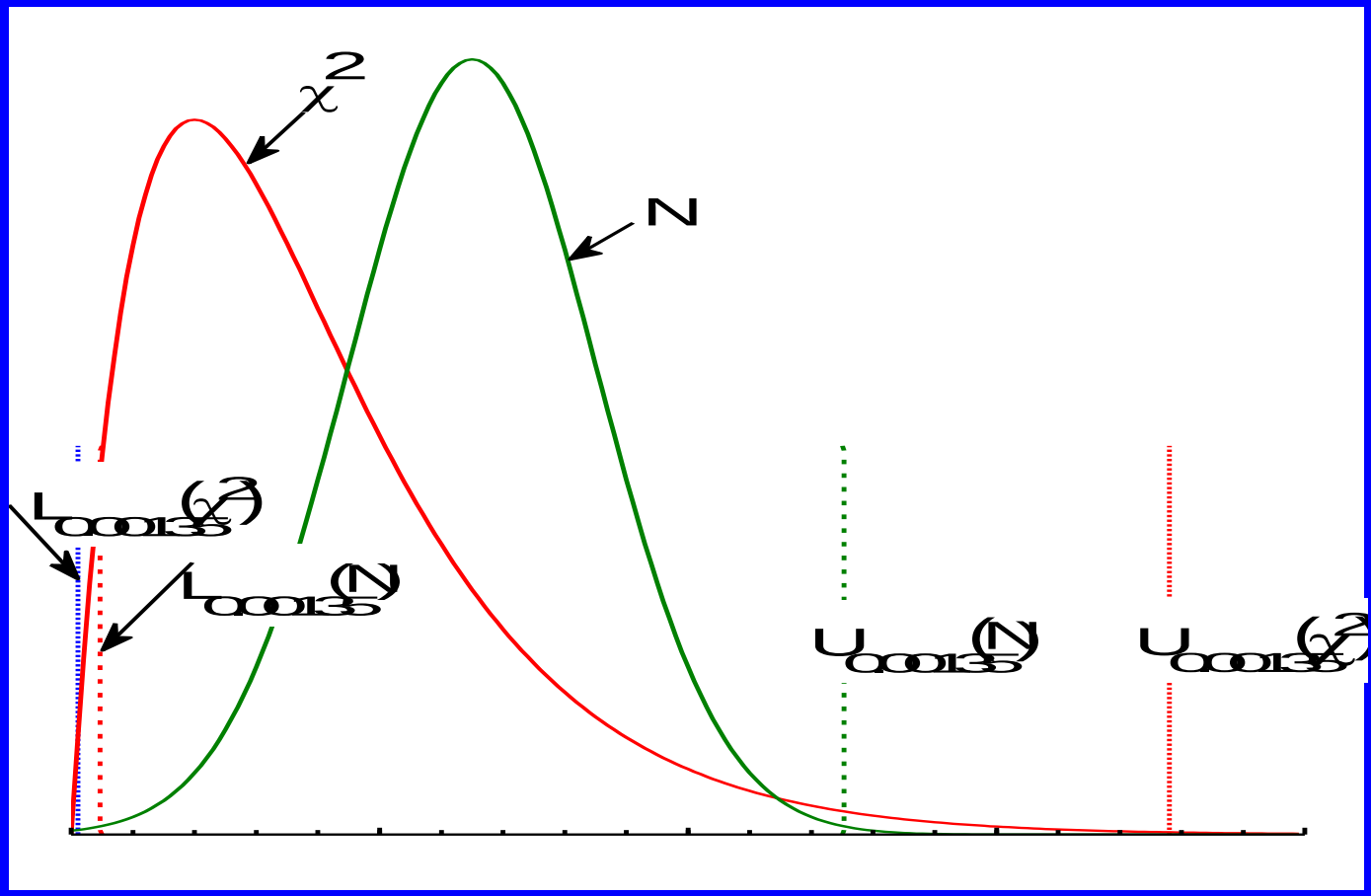


Example 12
Ampoule filling process
tolt.mtw

Stat>Capability Analysis (Normal)

USL=2.2
LSL=2.0




χ^2
N
$L_{0.00135}^{(\chi^2)}$
$L_{0.00135}^{(N)}$
$U_{0.00135}^{(N)}$
$U_{0.00135}^{(\chi^2)}$

$$C_P = \frac{USL - LSL}{U_p - L_p}$$

$$C_{PU} = \frac{USL - \mu_e}{U_p - \mu_e}$$

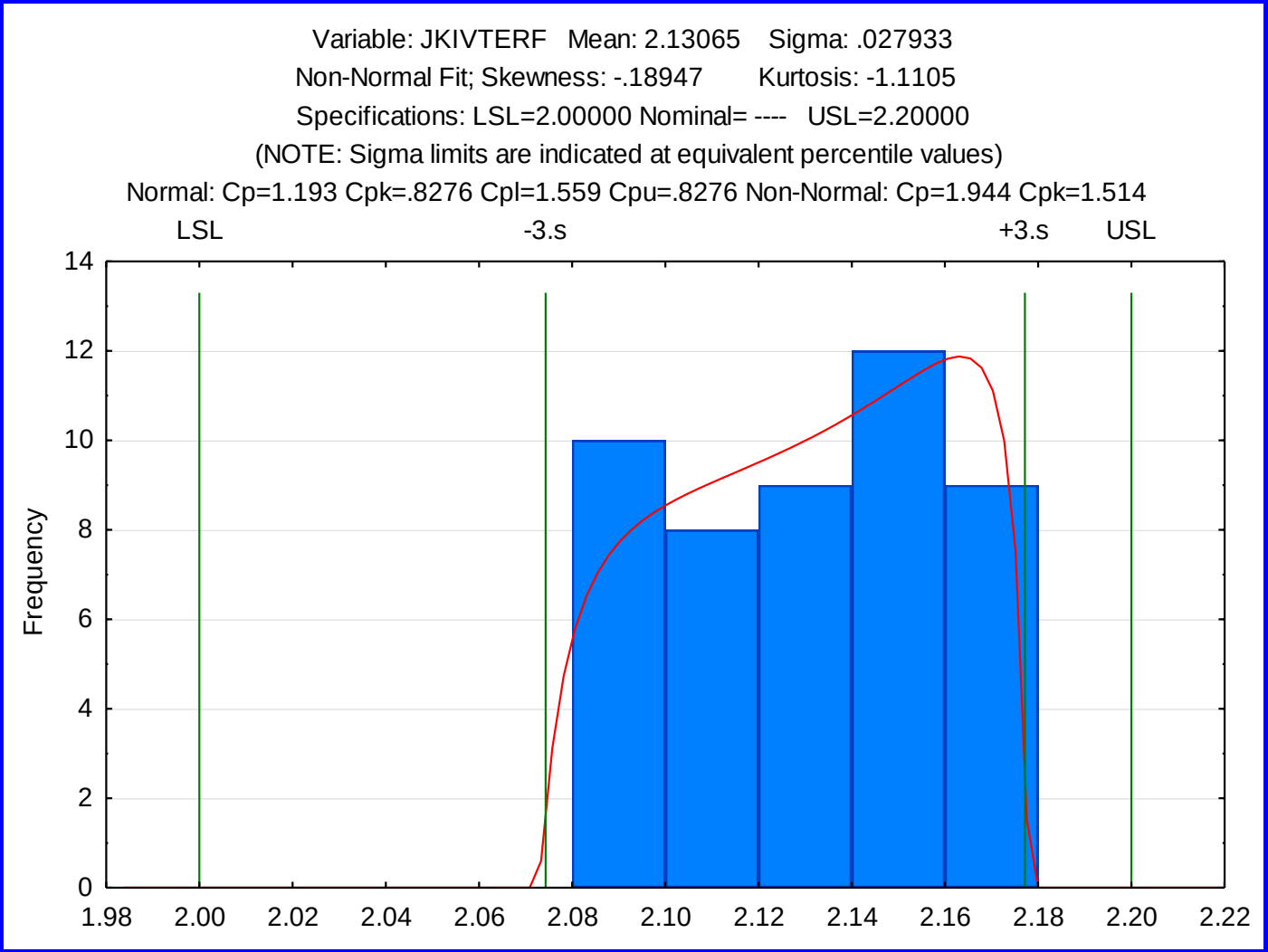
$$C_{PL} = \frac{\mu_e - LSL}{\mu_e - L_p}$$

μ_e the median,

U_p the critical value for P=0.99865,

L_p the critical value for P= 0.00135

ampoule filling process



The meaning of 6 sigma

In a 6 sigma process the proportion of defects is 3.4 ppm,

DPMO: Defect Per Million Opportunities

The capability analysis is meaningful only for a stable (in control) process.

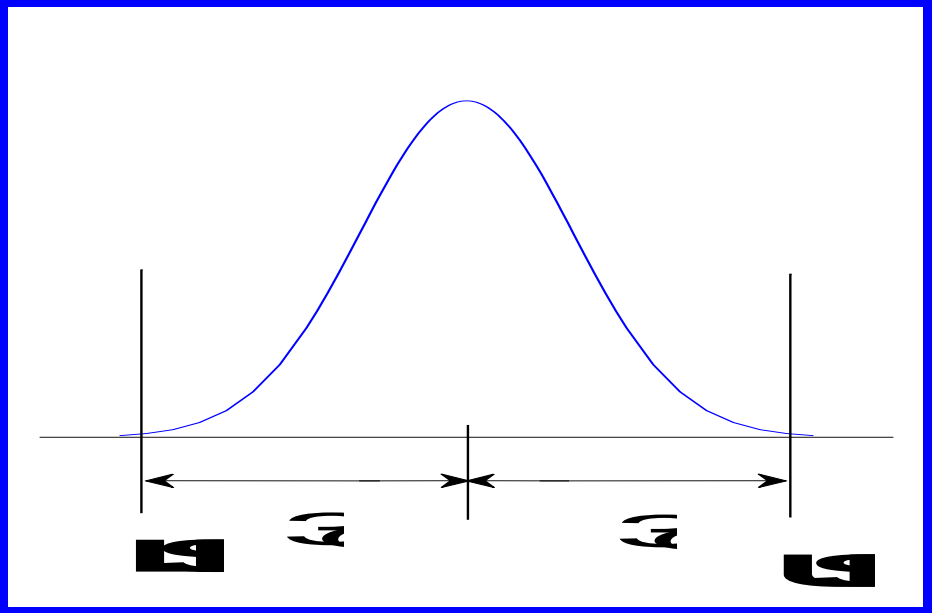
The two phases:

1. Stabilize the process for a reasonable time, removing the potential sources of variation (e.g. operator, batch of raw material)

2. Compare th long-term process capability with that expected

How to check the stability of the process?

The sigma capability measures the distance between the nominal value and the specification limit, in σ units.



$Z = 3$ $p=$

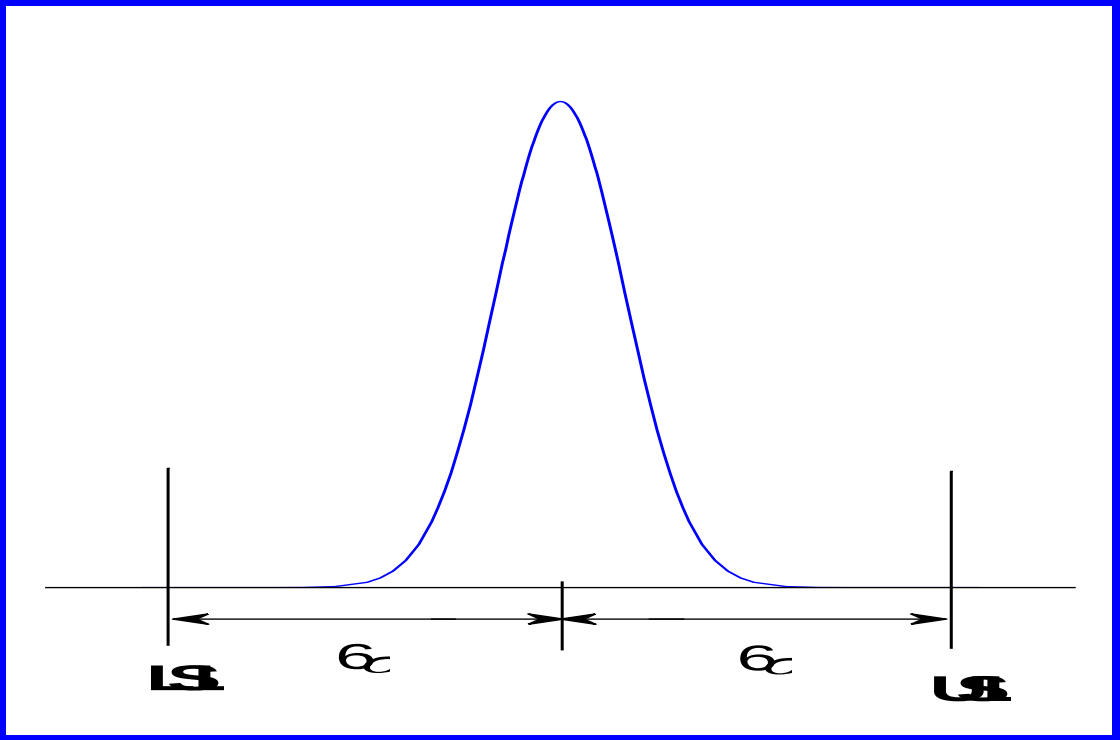
ppm

$C_P = C_{PK} =$

$$C_P = \frac{USL - LSL}{6\sigma}$$

If the process is centred:

$$C_P = C_{PK} = \frac{USL - T}{3\sigma}$$



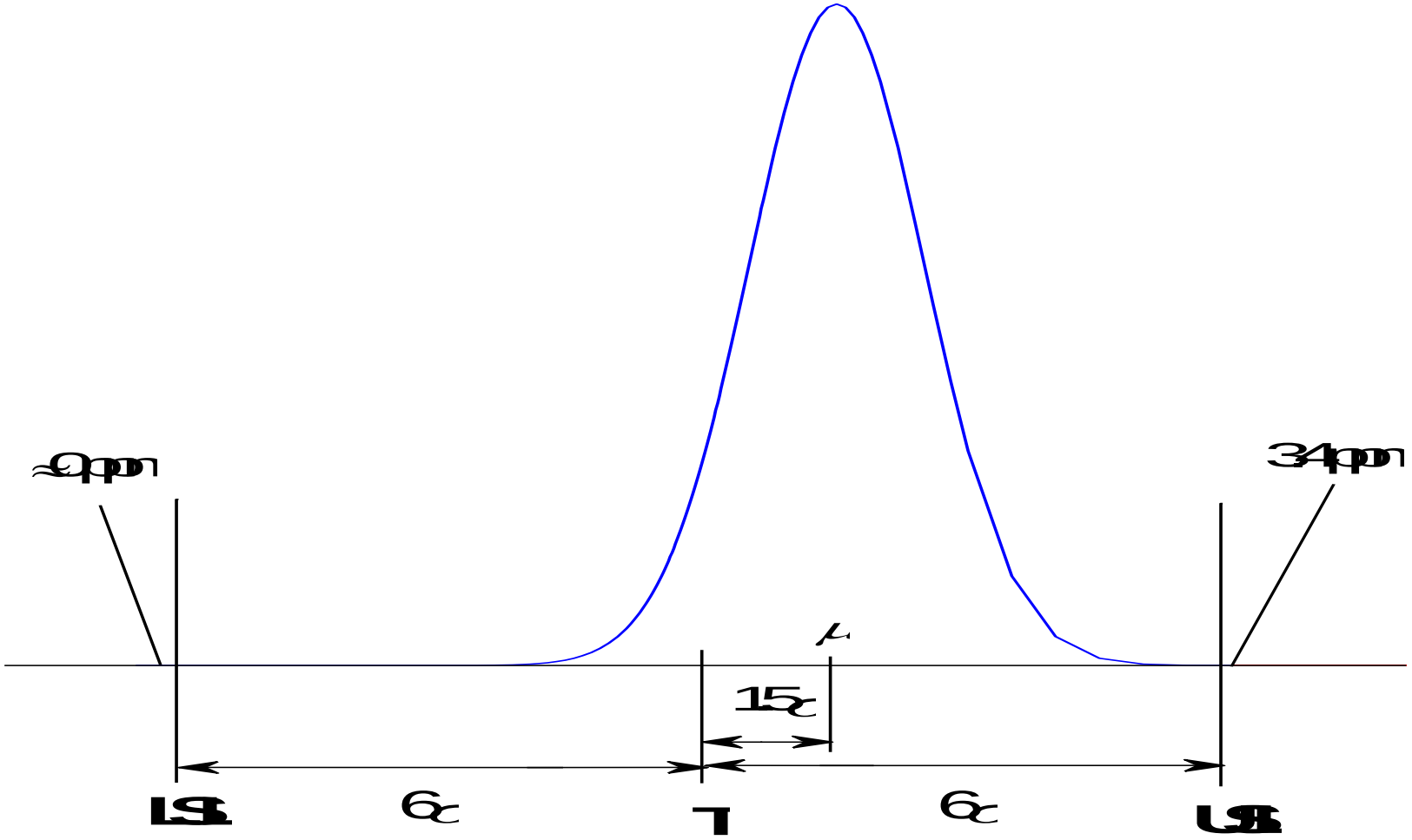
$Z = 6$

p= ppm

$C_P = C_{PK} =$

… if the process were centred (μ=T), but …

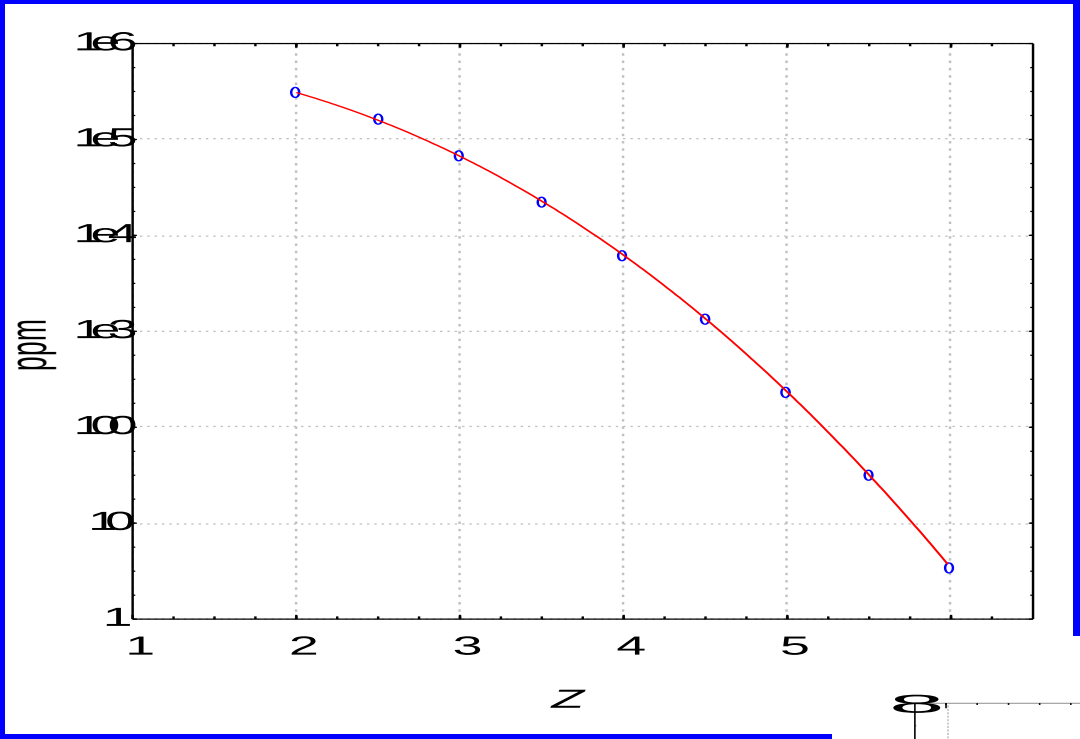
Motorola: 1.5σ shift

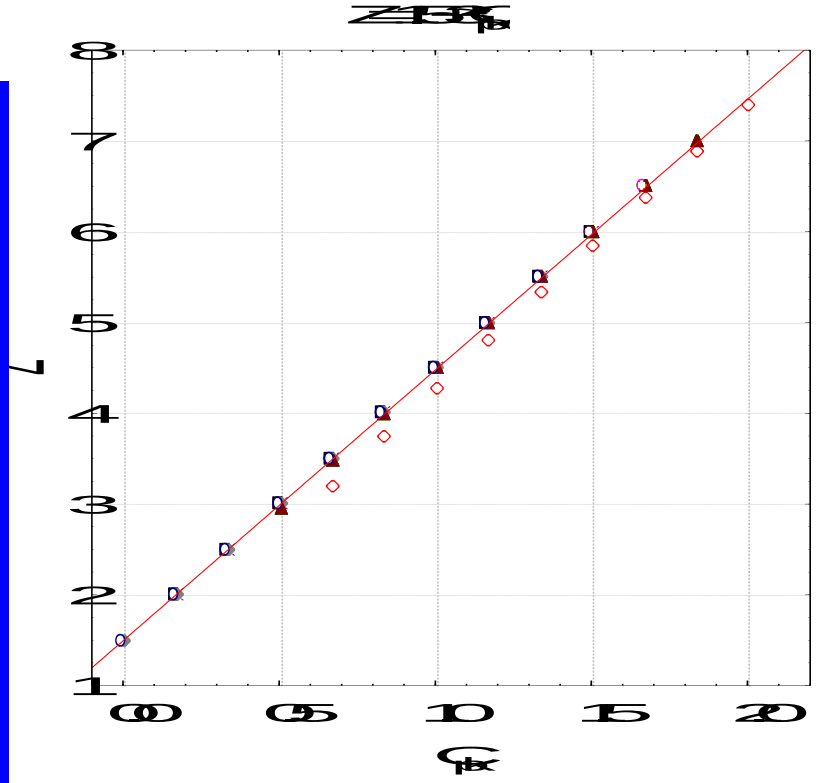


$C_P = 2$ $C_{PK} = 1.5$ Z=6 (process with 6 sigma capability)

sigma capability (Z)	C_P	C_{PK} with 1.5 σ shift	DPMO
2	0.67	0.17	308537
3	1	0.5	66807
4	1.33	0.67	6210
5	1.67	1.17	233
6	2	1.5	3.4

The sigma capability is a special scale for proportion of defects

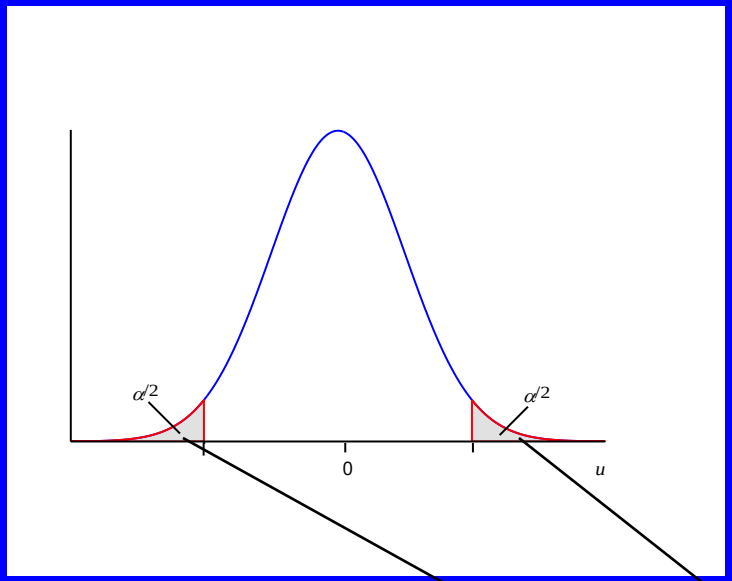

ppm
1e6
1e5
1e4
1e3
100
10
1
1
2
3
4
5
Z


8
7
6
5
4
3
2
1
0
5
10
15
20

If the shift differs from 1.5σ,

$$Z_{LT} + 1.5 \neq Z_{ST}$$

(LT: long term) (ST: short term)

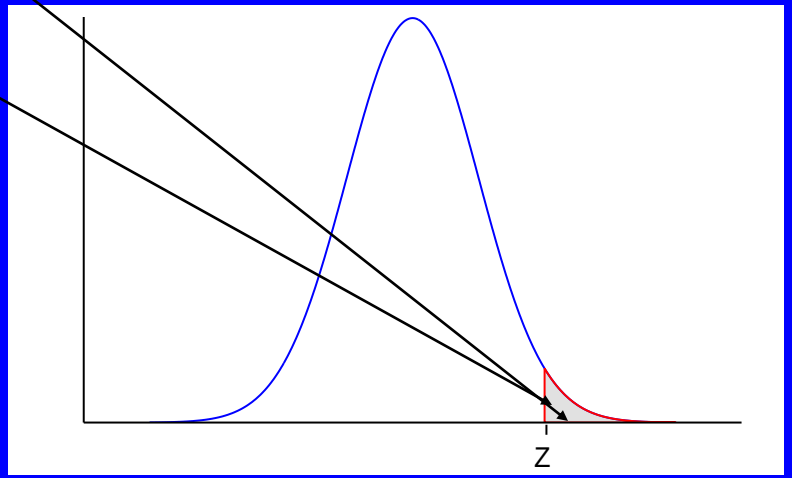


generalisation

do not care with the extent of shift

any distribution

$$Z = z + 1.5$$



Short term σ capability, alternative definitions

$$Z_{st} = \frac{SL - T}{\sigma_{st}}$$

where *SL* is *USL* or *LSL*
Minitab Six Sigma Process Report

This would be the short term σ capability of the process if it were properly centred

$$Z_{st} = \frac{SL - \mu}{\sigma_{st}}$$

Minitab Capability Analysis
Statistica Process Analysis

Example 9

Compute the sigma capability of data in YS3 column of cpdata1.sta file! Specification: 250±5 unit.

Process Capability Z-bench Statistics, Confidence Bounds etc (CPDATA1.STA)
Variable: YS3

Capability Index	Value
Cp - Lower CI	1.34213
Cp - Upper CI	1.83320
Cpk - Lower CI	1.13696
Cpk - Upper CI	1.57737
Z - benchmark Potential	4.07125
Z benchmark - LSL	5.45581
Z benchmark - USL	4.07149
Z benchmark - Lower CI	1.79578
Z benchmark - Upper CI	
Overall Process Performance	
PPM < LSL	0.02438
PPM > USL	23.35658
PPM Total	23.38095
Observed Process Performance	
PPM < LSL	0.00000
PPM > USL	0.00000
PPM Total	0.00000
Cpm - Lower CI	0.97565
Cpm - Upper CI	1.27832

Variable: YS3 (CPDATA1.STA)
-3.000 *Sigma=247.578
+3.000 *Sigma=253.875

Capability Index	Value
Lower Specification Limit	245.000
Nominal Specification	250.000
Upper Specification Limit	255.000
CP (potential capability)	1.5879
CR (capability ratio)	0.6298
CPK (demonstrated excellence)	1.3572
CPL (lower capability index)	1.8186
CPU (upper capability index)	1.3572
K (non-centering correction)	0.1453
CPM (potential capability II)	1.1271

(Statistica Process Analysis)

$$Z_{st} = \frac{SL - \mu}{\sigma_{st}}$$

short term

$$Z_{USL} = \frac{255 - 250.7265}{1.0496} = 4.071$$

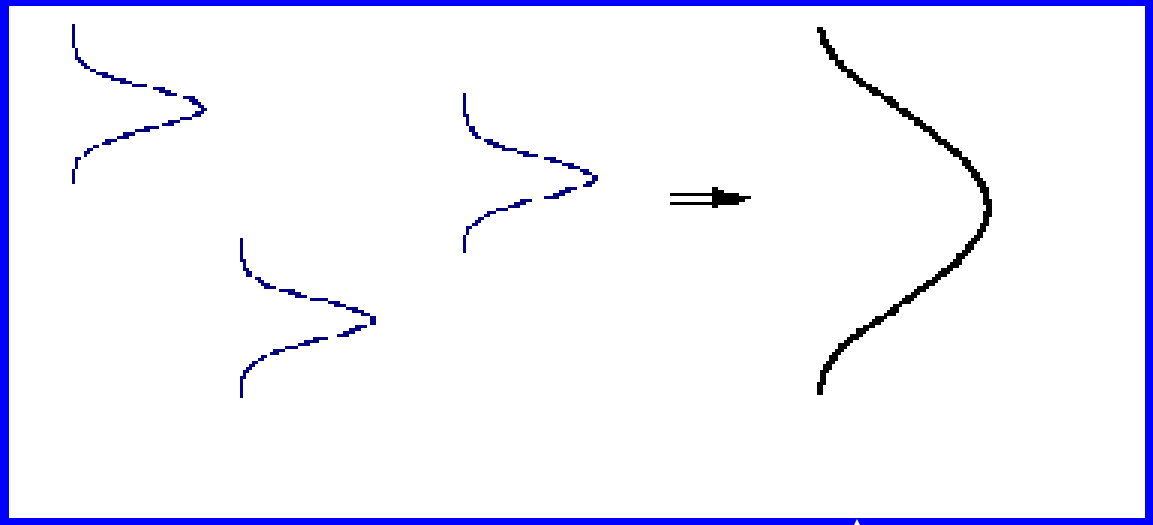
$$P(z > Z_{USL}) = 23.5\text{ppm}$$

$$Z_{LSL} = \frac{250.7265 - 245}{1.0496} = 5.456$$

$$P(z < Z_{LSL}) = 0.024\text{ppm}$$

$$P_{total} = 23.52\text{ppm}$$

$$Z_{st} = 4.071$$



σ_{ST}
(short term)

σ_{LT}
(long term)

Long term σ capability

$$Z_{lt} = \frac{SL - \mu}{\sigma_{lt}}$$

	Process Capability Z-bench Statistics, Confidence Bounds etc (CPDATA1.STA) Variable: YS3
	Value
Pp - Lower CI	1.115754E+00
Pp - Upper CI	1.476234E+00
Ppk - Lower CI	9.402471E-01
Ppk - Upper CI	1.275380E+00
Z - benchmark Overall	3.320802E+00
Z benchmark - LSL	4.453421E+00
Z benchmark - USL	3.323441E+00
Z benchmark - Lower CI	1.785034E+00
Z benchmark - Upper CI	-3.000000E+30
Potential Process Performance	
PPM < LSL	4.225645E+00
PPM > USL	4.445709E+02
PPM Total	4.487965E+02

	Variable: YS3 (CPDATA1.STA) -3.000 *Sigma=246.869 +3.000 *Sigma=254.584
	Value
Lower Specification Limit	245.0000
Nominal Specification	250.0000
Upper Specification Limit	255.0000
PP (performance index)	1.2961
PR (performance ratio)	0.7715
PPK (perf. demonstr. excell.)	1.1078
PPL (lower performance index)	1.4845
PPU (upper performance index)	1.1078

(Statistica Process Analysis)

long term

$$Z_{lt} = \frac{SL - \mu}{\sigma_{lt}}$$

$$Z_{USL} = \frac{255 - 250.7265}{1.286} = 3.323 \qquad P(z > Z_{USL}) = 445\text{ppm}$$

$$Z_{LSL} = \frac{250.7265 - 245}{1.286} = 4.452 \qquad P(z < Z_{LSL}) = 4.3\text{ppm}$$

$$P_{total} = 449\text{ppm} \qquad Z_{lt} = 3.32$$

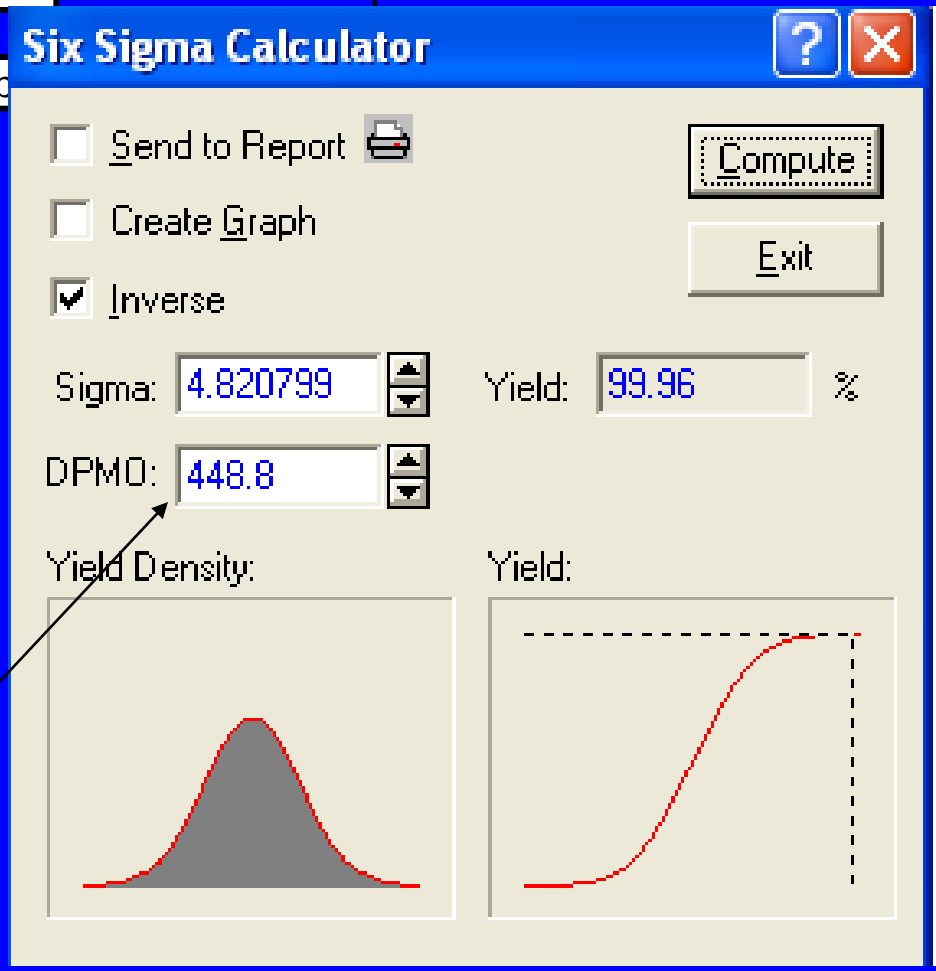
$$Z_{st} = 4.07 \qquad Z_{shift} = 4.07 - 3.32 = 0.75$$

Variable: YS3, Distribution: Normal (cpdata1)
Specifications: Lower=245.000 Nominal=250.000 Upper=255.000
Mean:250.73,Std.Dv:1.2859

	Observed	Percent Observed	Expected	Percent Expected
Above USL:	0	0.00	0.044457	0.044457
Below LSL:	0	0.00	0.000423	0.000423
Total	0	0.00	0.044880	0.044880

3.32+1.5=4.82

	Process Capability Z-b
	Value
Pp - Lower CI	1.115754E+00
Pp - Upper CI	1.476234E+00
Ppk - Lower CI	9.402471E-01
Ppk - Upper CI	1.275380E+00
Z - benchmark Overall	3.320802E+00
Z benchmark - LSL	4.453421E+00
Z benchmark - USL	3.323441E+00
Z benchmark - Lower CI	1.785034E+00
Z benchmark - Upper CI	-3.000000E+30
Potential Process Performance	
PPM < LSL	4.225645E+00
PPM > USL	4.445709E+02
PPM Total	4.487965E+02



with the alternative short term sigma capability definition:

$$Z_{st} = \frac{SL - T}{\sigma_{st}}$$

$$Z_{st} = \frac{255 - 250}{1.0496} = 4.76$$

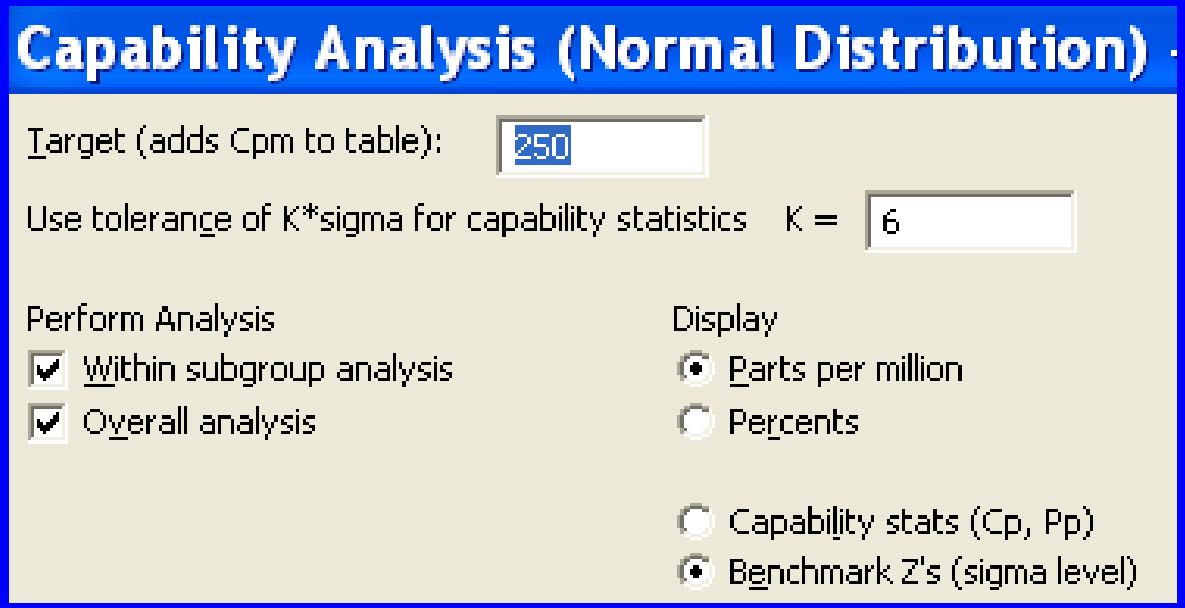
$$Z_{lt} = 3.32$$

$$Z_{shift} = 4.76 - 3.32 = 1.44$$

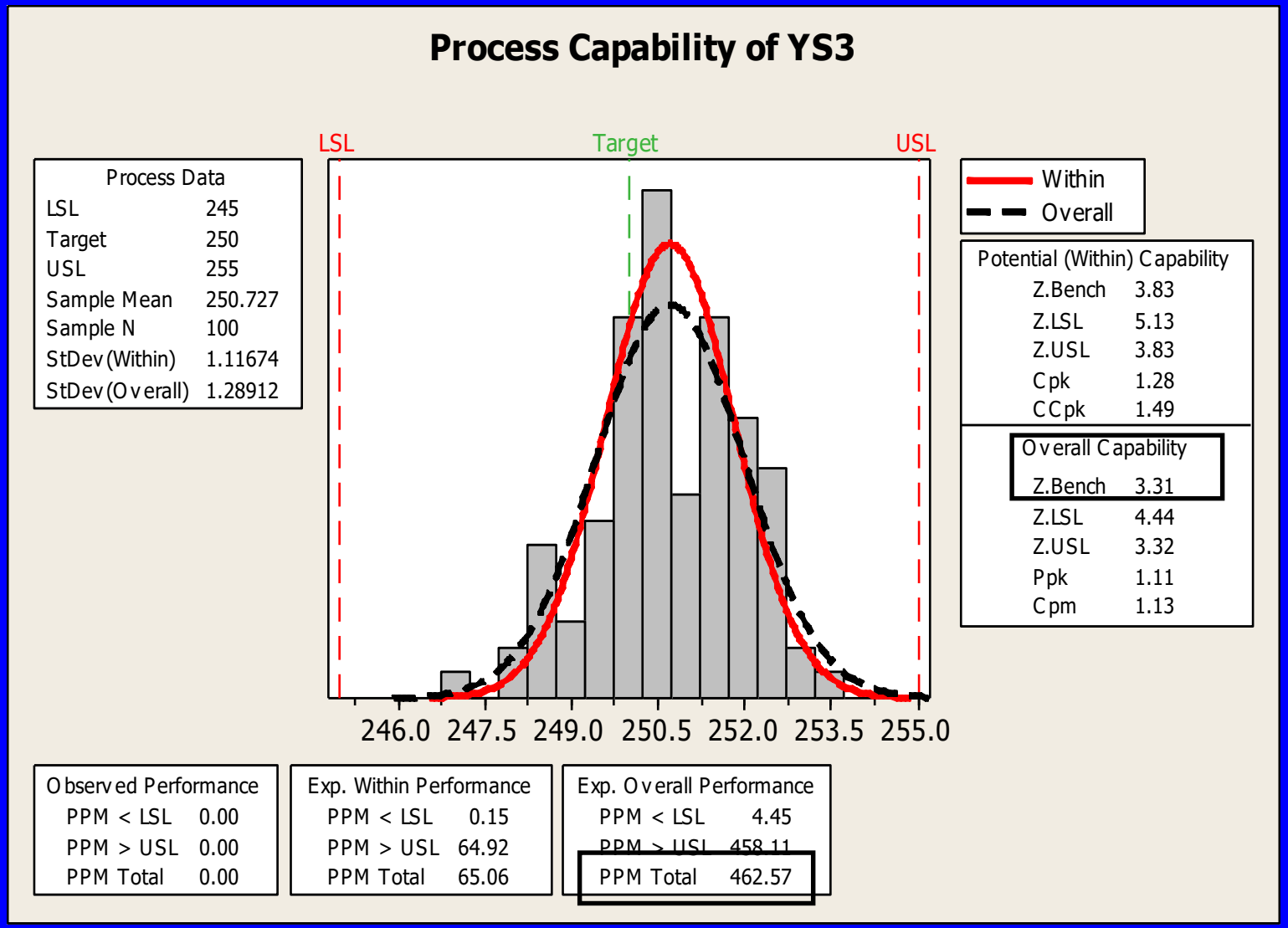
Example 15

Compute the sigma capability of data in YS3 column of cpdata1.mtw file!

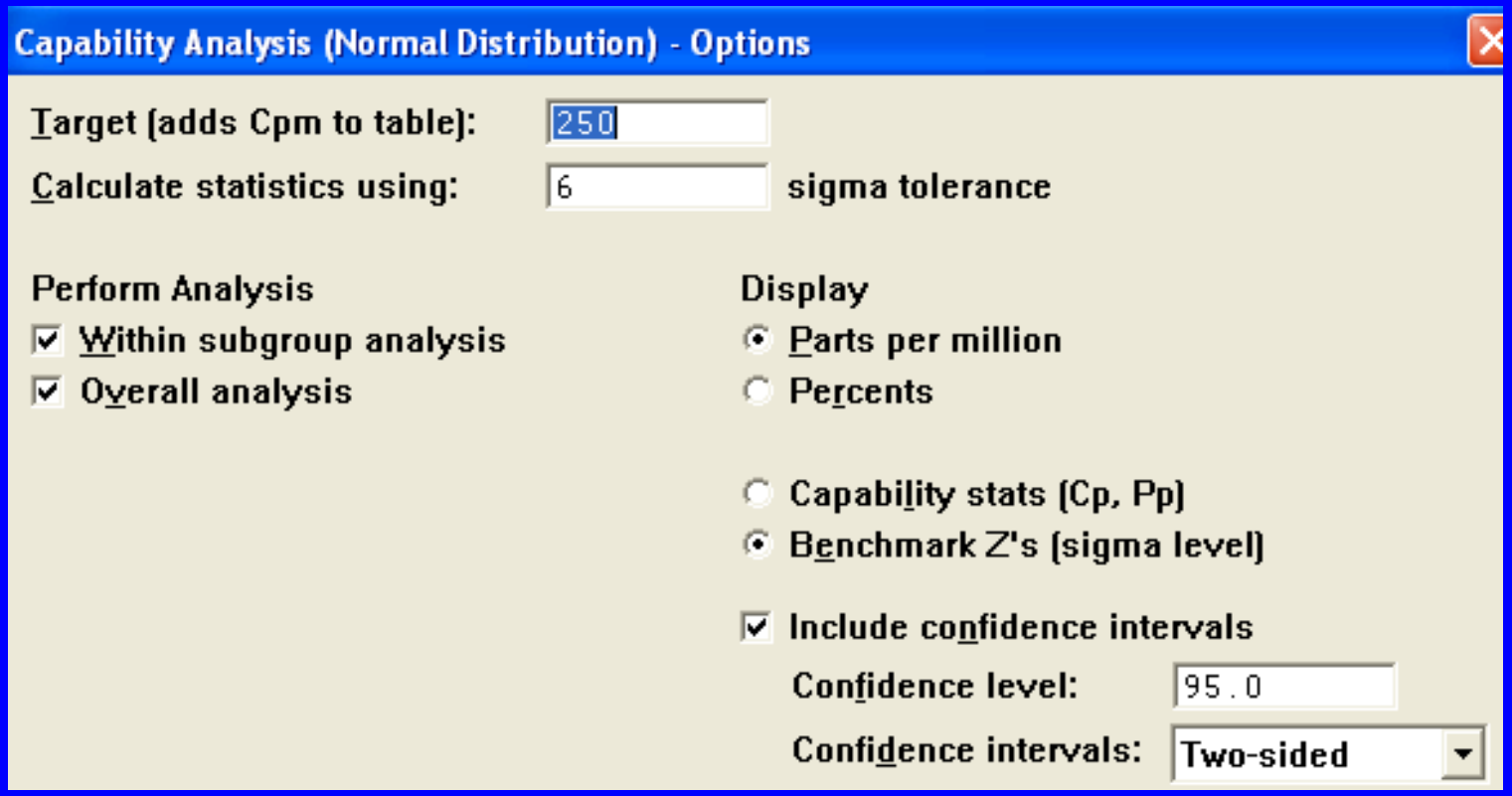
Specification: 250±5 unit.



Options: benchmark Z's Szigma képesség: 3.32+1.5=4.82

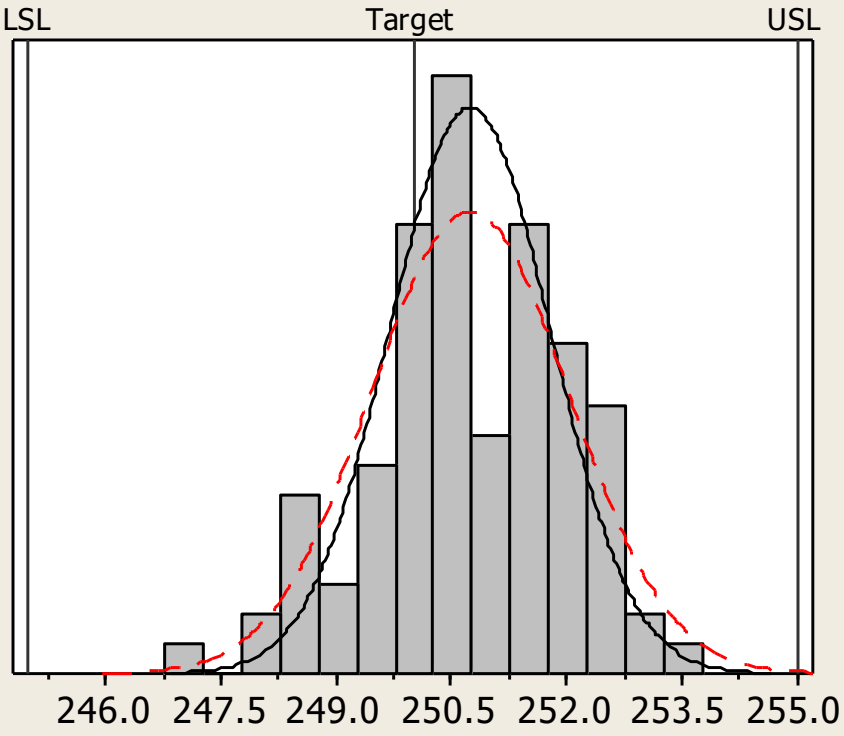


The Z value (as C_P, C_{PK}) computed from sample data are random variables, give their confidence interval as well.!



Process Capability of YS3

(using 95.0% confidence)



Process Data	
LSL	245.00000
Target	250.00000
USL	255.00000
Sample Mean	250.72650
Sample N	100
StDev(Within)	1.05290
StDev(Overall)	1.28912

Within
Overall

Potential (Within) Capability	
Z.Bench	4.06
Lower CL	1.80
Upper CL	*
Z.LSL	5.44
Z.USL	4.06
Cpk	1.35
Lower CL	1.13
Upper CL	1.57
CCpk	1.58

Overall Capability	
Z.Bench	3.31
Lower CL	1.78
Upper CL	*
Z.LSL	4.44
Z.USL	3.32
Ppk	1.11
Lower CL	0.94
Upper CL	1.27
Cpm	1.13
Lower CL	0.98

Observed Performance	
PPM < LSL	0.00
PPM > USL	0.00
PPM Total	0.00

Exp. Within Performance	
PPM < LSL	0.03
PPM > USL	24.66
PPM Total	24.69

Exp. Overall Performance	
PPM < LSL	4.45
PPM > USL	458.11
PPM Total	462.57

MTB14

Calculation of sigma capability:

DPMO $\rightarrow z_{LT} \rightarrow Z = z + 1.5$

$$Z_{lt} = \frac{SL - \mu}{\sigma_{lt}}$$

Special case $\quad \mu = T + 1.5\sigma_{ST} \qquad \sigma_{ST} = \sigma_{LT}$

$$Z = \frac{USL - T}{\sigma_{ST}} = \frac{USL - \mu + 1.5\sigma_{ST}}{\sigma_{ST}} = z + 1.5 \leftarrow Z_{shift}$$

General case $\quad Z_{shift} = \frac{USL - T}{\sigma_{ST}} - \frac{USL - \mu}{\sigma_{ST}} = \frac{\mu - T}{\sigma_{ST}} = Z - z$

$$Z_{shift} = \frac{USL - T}{\sigma_{ST}} - \frac{USL - \mu}{\sigma_{LT}} = Z - z$$

The process capability study is to be interpreted for in-control processes only.

Two parts of the task:

1. Stabilize the process for an acceptable time span, eliminating potential sources of fluctuation (e.g. operator, lot of raw material)

2. Compare the long term process performance with that expected

How to check stability?