MEASUREMENT SYSTEM ANALYSIS

R&R study

The purpose is to check if the error in measurement system is small enough to get reliable data from the process studied.

Variables data
(interval and proportional scale: ^{0}C, kg, N)

Attribute data
(nominal and ordinal scale: good/bad, stage, rank)

Variables data

bias (accuracy)

precision (R&R)

- repeatability
- reproducibility by different operators
- ratio of precision (measurement error) to the variation between parts
- estimation of variance components

Accuracy (bias)

$$E(x) = x_{ref}$$

x_{ref} : standard

$$\mathrm{H}_0 : \; E(x) = x_{ref}$$

one-sample t test

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

H_0 (no bias) is accepted at α significance level if

$$P\left(-t_{a/2} < \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \le t_{a/2} \right) = 1 - \alpha$$

Example

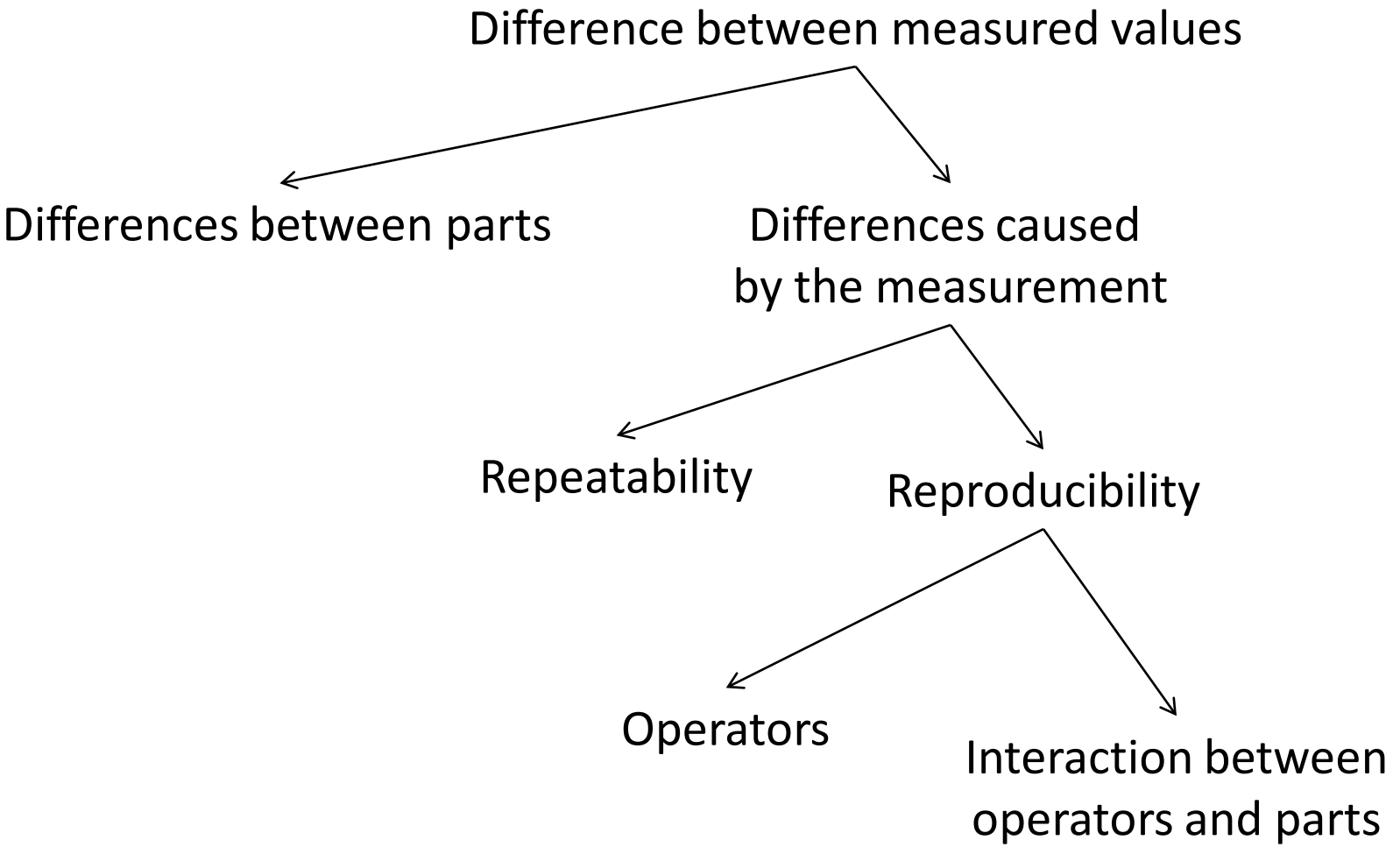
$$H_0 : E(x) = x_{ref}$$ x_{ref}=6.0 (standard)

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

i	x_i	$x_i - x_{ref}$
1	5.8	-0.2
2	5.7	-0.3
3	5.9	-0.1
4	5.9	-0.1
5	6.0	0.0
6	6.1	0.1
7	6.0	0.0
8	6.1	0.0
9	6.4	0.4
10	6.3	0.3
11	6.0	0.0
12	6.1	0.1
13	6.2	0.2
14	5.6	-0.4
15	6.0	0.0

Variable	Test of means against reference constant (value) (gagebias)							
	Mean	Std.Dv.	N	Std.Err.	Reference Constant	t-value	df	p
x	6.006667	0.212020	15	0.054743	6.000000	0.121781	14	0.904804

Splitting the differences into components



Total variance of measurement data :

$$\sigma^2_{\text{total}} = \sigma^2_{\text{parts}} + \sigma^2_{\text{R\&R}}$$

Fluctuation attributable to the measurement (precision):

$$\sigma^2_{\text{R\&R}} = \sigma^2_{\text{reprod}} + \sigma^2_{\text{repeat}}$$

Reproducibility:

$$\sigma^2_{\text{reprod}} = \sigma^2_{\text{oper}} + \sigma^2_{\text{part*oper}}$$

Design of experiments for the study

A certain number (e.g. 10) is selected randomly from among the parts produced by the process to be investigated, all of them measured several (e.g. 3) times by each of the selected operators (e.g. 4).

operator	A			B			C		
part	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3	rept 1	rept 2	rept 3
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									

Results:

- The variance components are related to the total variance.
- Analogously to the C_P process capability index the ranges attributed to the variance components is related to the width of the spec. range (P/T precision to tolerance) . Actually the 99% (5.15 σ width) interval is in the numerator:

$$\frac{P}{T} = \frac{5.15 \cdot \hat{\sigma}_{\mathrm{R\&R}}}{USL - LSL}.$$

6.0 may stand for 5.15, expressing the $\pm 3\sigma$ limit (99.73% instead of 99%)

Number of distinguishable categories (discrimination index)

$$\frac{\hat{\sigma}_{\mathrm{part}}}{\hat{\sigma}_{\mathrm{R\&R}}}\sqrt{2}$$

rounded down to integer

Variance estimation: Range method

Variances are estimated from ranges, e.g.

$$\hat{\sigma}_{\text{repeat}} = \frac{\overline{R}_{\text{repeat}}}{d_2}$$

$\overline{R}_{\text{repeat}}$ is the average range of repetitions

d_2 is taken from a Table for the # of repetitions

Similarly for $\hat{\sigma}_{\text{reprod}}$ $\hat{\sigma}_{\text{part}}$

for small sample sizes different d_2 values apply

Variance estimation: ANOVA method

The model (two-way cross-classification with random factors, repeated measurements)

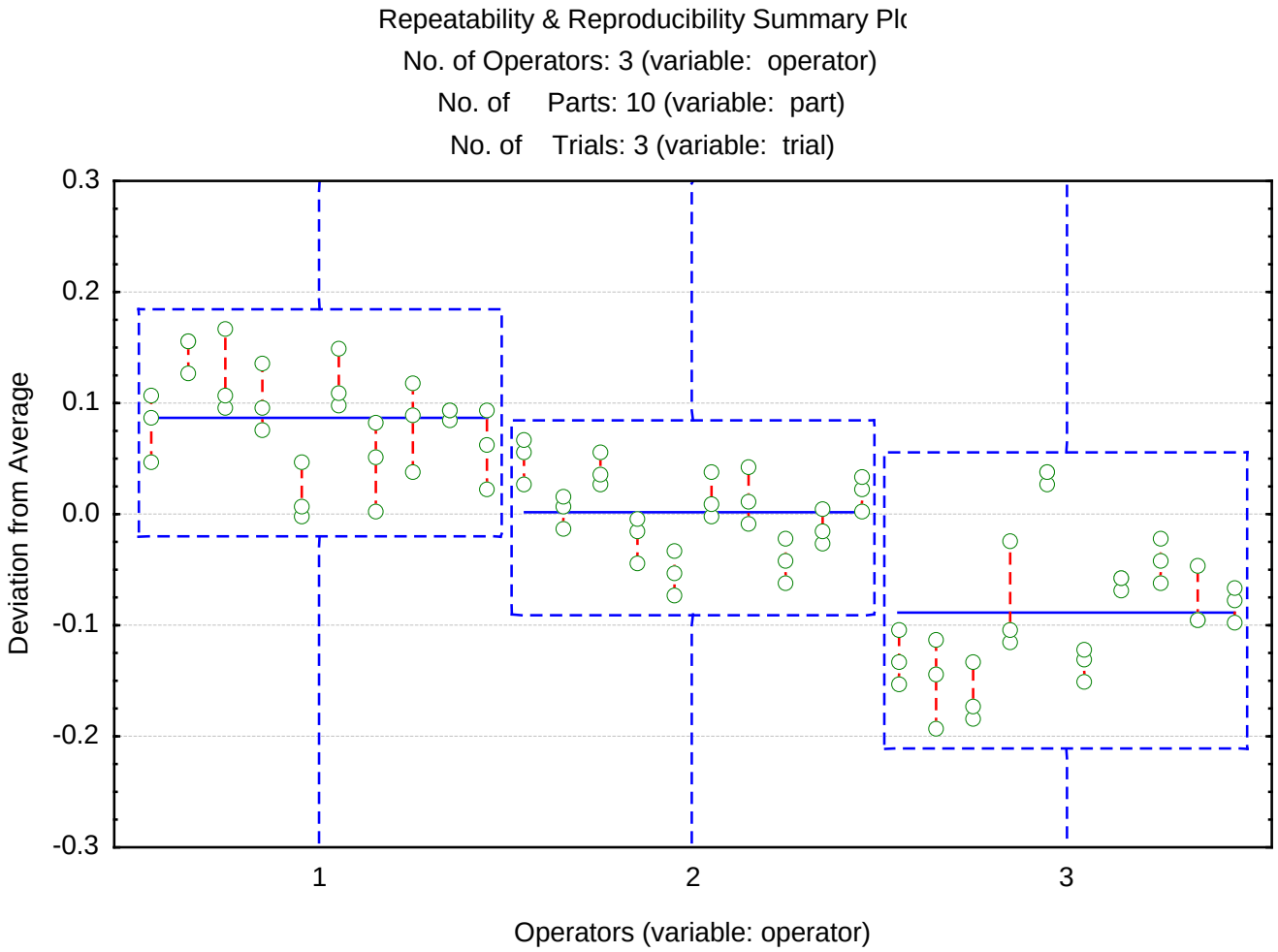
$$x_{ijk} = \mu + P_i + O_j + PO_{ij} + \varepsilon_{k(ij)}$$

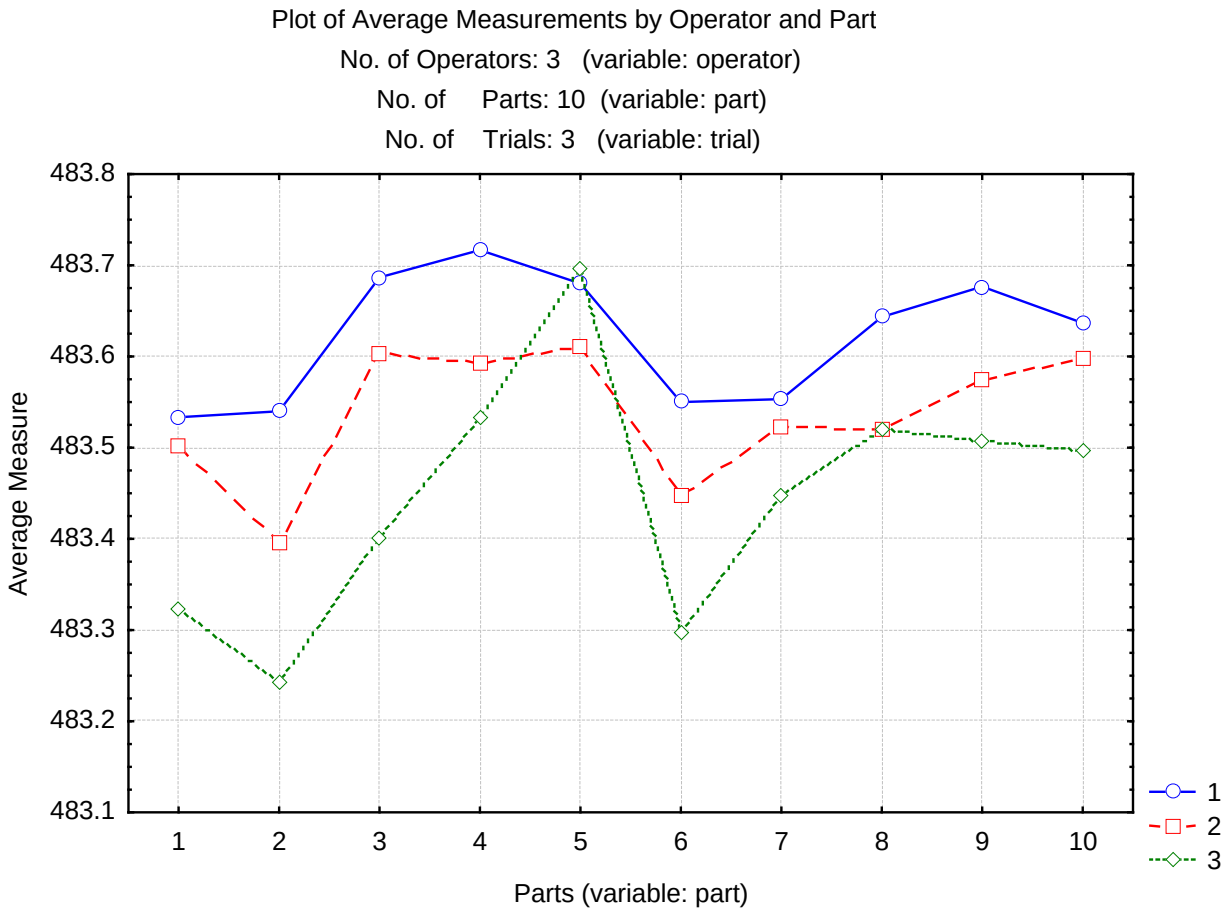
P is for parts
O is for operators
ε experimental error

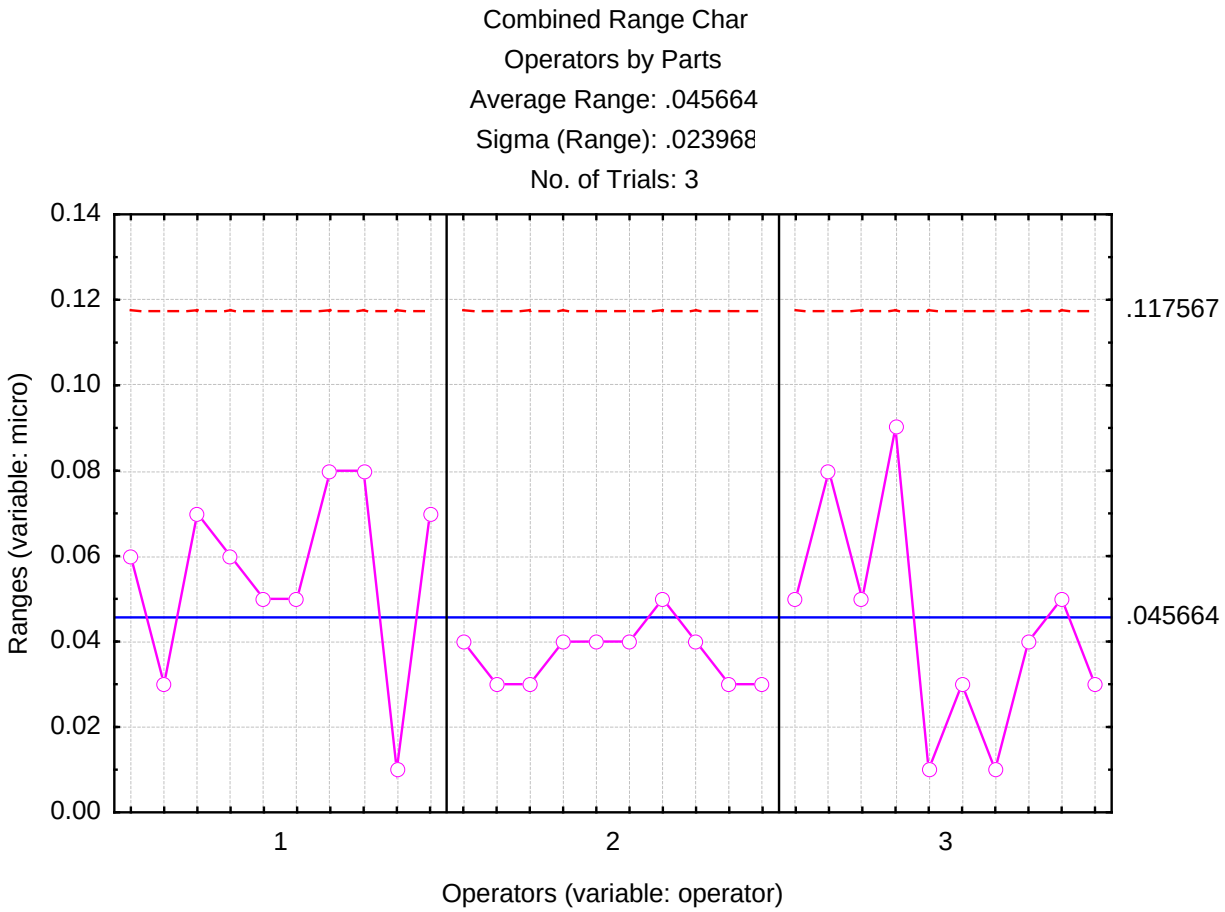
Example

The width of the specification for the inner diameter 1.52 mm.
10 parts are taken randomly from the manufacturing, each of them are measured 3 times by 3 operators.

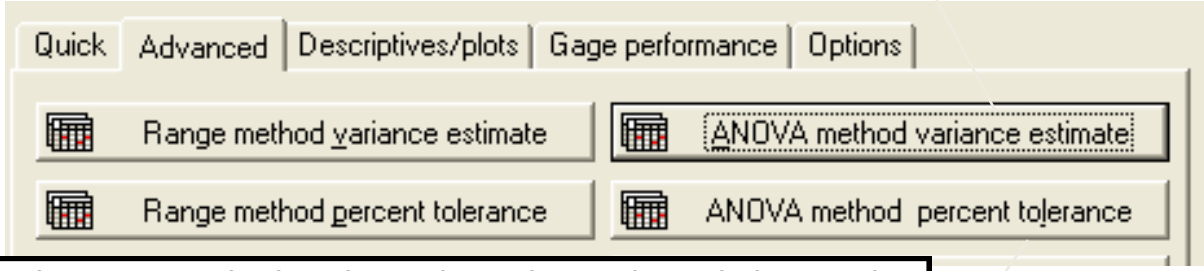
Perform a Gauge R&R study!


Repeatability & Reproducibility Summary Plc
No. of Operators: 3 (variable: operator)
No. of Parts: 10 (variable: part)
No. of Trials: 3 (variable: trial)
Deviation from Average
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
1
2
3
Operators (variable: operator)


Plot of Average Measurements by Operator and Part
No. of Operators: 3 (variable: operator)
No. of Parts: 10 (variable: part)
No. of Trials: 3 (variable: trial)
Average Measure
483.8
483.7
483.6
483.5
483.4
483.3
483.2
483.1
1
2
3
4
5
6
7
8
9
10
Parts (variable: part)
1
2
3


Combined Range Char
Operators by Parts
Average Range: .045664
Sigma (Range): .023968
No. of Trials: 3
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Ranges (variable: micro)
.117567
.045664
1
2
3
Operators (variable: operator)

Source (Expected MS)	Variance Components; Variable: micro (micro.sta) Mean=483.535 Std.Dv=.119260 Operators: 3 Parts: 10 Trials: 3 Estimatd Sigma	.90 Lowr Conf.Lim	.90 Uppr Conf.Lim	Estimatd Variance	% of R & R	% of Total
Repeatability	0.026055	0.022695	0.030711	0.000679	6.2345	3.9321
Operator	0.085921	0.043097	0.386196	0.007382	67.7951	42.7588
Interaction (OP)	0.053179	0.038615	0.077027	0.002828	25.9704	16.3797
Part-to-Part	0.079849	0.044634	0.144458	0.006376		36.9294
Combined R & R	0.104352	0.075945	0.391256	0.010889	100.0000	63.0706
Total	0.131397			0.017265		100.0000



Source (Expected MS)	Percent Tolerance Analysis:micro Sigma intervals:6. (micro.sta) Mean=483.535 Std.Dv=.119260 Operators: 3 Parts: 10 Trials: 3 Measrmnt Units	.90 Lowr Conf.Lim	.90 Uppr Conf.Lim	% Proc. Variatn	% Total Contrib.	% of Tolernce
Repeatability (Equipment Var).	0.156333	0.136172	0.184266	19.8296	3.9321	10.2851
Operator (Appraiser Var.)	0.515525	0.258584	2.317176	65.3902	42.7588	33.9161
Interaction (Operator x Part)	0.319073	0.231688	0.462160	40.4718	16.3797	20.9916
Part Variation	0.479096	0.267801	0.866750	60.7695	36.9294	31.5195
Combined R & R	0.626110	0.455672	2.347534	79.4170	63.0706	41.1914
Total Process Variation	0.788382			100.0000	100.0000	51.8673
Tolerance	1.520000					100.0000